



# The Fibonacci Quarterly

THE OFFICIAL JOURNAL OF THE FIBONACCI ASSOCIATION

## TABLE OF CONTENTS

Convergence of $r$ -Generalized Fibonacci Sequences and an Extension of Ostrowski's Condition .....	<i>R.B. Taher, M. Mouline and M. Rachidi</i>	386
A Note on a Class of Computational Formulas Involving the Multiple Sum of Recurrence Sequences .....	<i>Zhizheng Zhang and Xin Wang</i>	394
The Passing of Three Fibonacci Association Friends .....		398
Eulerian Polynomials and Related Explicit Formulas .....	<i>Ching-Hua Chang and Chung-Wei Ha</i>	399
Author and Title Index .....		404
Reflections on the Lambda Triangle .....	<i>A.G. Shannon and A.F. Horadam</i>	405
Report on the Tenth International Conference on Fibonacci Numbers and Their Applications .....		416
Combinatorial Matrices and Linear Recursive Sequences .....	<i>A.G. Shannon and R.L. Ollerton</i>	417
On Chebyshev Polynomials and Fibonacci Numbers .....	<i>Wenpeng Zhang</i>	424
On the Number of Permutations within a Given Distance .....	<i>O. Krafft and M. Schaefer</i>	429
Generalization of a Theorem of Drobot .....	<i>Lawrence Somer</i>	435
On the Product of Line-Sequences .....	<i>Jack Y. Lee</i>	438
Fibonacci Tree Is Critically Balanced—A Note .....	<i>Yasuichi Horibe</i>	441
Generalized Fibonacci Sequences and Linear Congruences .....	<i>Giovanni Sburlati</i>	446
Application of Markov Chains Properties to $\infty$ -Generalized Fibonacci Sequences .....	<i>F. Chaoui, M. Mouline and M. Rachidi</i>	453
Perfect Squares in the Lucas Numbers .....	<i>Minoru Yabuta</i>	460
Elementary Problems and Solutions .....	<i>Edited by Russ Euler and Jawad Sadek</i>	467
Advanced Problems and Solutions .....	<i>Edited by Raymond E. Whitney</i>	472
Volume Index .....		479

VOLUME 40

NOVEMBER 2002

NUMBER 5

## PURPOSE

The primary function of **THE FIBONACCI QUARTERLY** is to serve as a focal point for widespread interest in the Fibonacci and related numbers, especially with respect to new results, research proposals, challenging problems, and innovative proofs of old ideas.

## EDITORIAL POLICY

**THE FIBONACCI QUARTERLY** seeks articles that are intelligible yet stimulating to its readers, most of whom are university teachers and students. These articles should be lively and well motivated, with new ideas that develop enthusiasm for number sequences or the exploration of number facts. Illustrations and tables should be wisely used to clarify the ideas of the manuscript. Unanswered questions are encouraged, and a complete list of references is absolutely necessary.

## SUBMITTING AN ARTICLE

Articles should be submitted using the format of articles in any current issues of **THE FIBONACCI QUARTERLY**. They should be typewritten or reproduced typewritten copies, that are clearly readable, double spaced with wide margins and on only one side of the paper. The full name and address of the author must appear at the beginning of the paper directly under the title. Illustrations should be carefully drawn in India ink on separate sheets of bond paper or vellum, approximately twice the size they are to appear in print. Since the Fibonacci Association has adopted  $F_1 = F_2 = 1$ ,  $F_{n+1} = F_n + F_{n-1}$ ,  $n \geq 2$  and  $L_1 = 1$ ,  $L_2 = 3$ ,  $L_{n+1} = L_n + L_{n-1}$ ,  $n \geq 2$  as the standard definitions for The Fibonacci and Lucas sequences, these definitions *should not* be a part of future papers. However, the notations *must* be used. One to three *complete* A.M.S. classification numbers *must* be given directly after references or on the bottom of the last page. **Papers not satisfying all of these criteria will be returned.** See the worldwide web page at:

<http://www.sdstate.edu/~wesc/http/fibhome.html>.

Three copies of the manuscript should be submitted to: **CURTIS COOPER, DEPARTMENT OF MATHEMATICS AND COMPUTER SCIENCE, CENTRAL MISSOURI STATE UNIVERSITY, WARRENSBURG, MO 64093-5045.**

Authors are encouraged to keep a copy of their manuscripts for their own files as protection against loss. The editor will give immediate acknowledgment of all manuscripts received.

The journal will now accept articles via electronic services. However, electronic manuscripts must be submitted using the typesetting mathematical wordprocessor AMS-TeX. Submitting manuscripts using AMS-TeX will speed up the refereeing process. AMS-TeX can be downloaded from the internet via the homepage of the American Mathematical Society.

## SUBSCRIPTIONS, ADDRESS CHANGE, AND REPRINT INFORMATION

Address all subscription correspondence, including notification of address change, to: **PATTY SOLSAA, SUBSCRIPTIONS MANAGER, THE FIBONACCI ASSOCIATION, P.O. BOX 320, AURORA, SD 57002-0320. E-mail: [solsaap@itctel.com](mailto:solsaap@itctel.com).**

Requests for reprint permission should be directed to the editor. However, general permission is granted to members of The Fibonacci Association for noncommercial reproduction of a limited quantity of individual articles (in whole or in part) provided complete reference is made to the source.

Annual domestic Fibonacci Association membership dues, which include a subscription to **THE FIBONACCI QUARTERLY**, are \$40 for Regular Membership, \$50 for Library, \$50 for Sustaining Membership, and \$80 for Institutional Membership; foreign rates, which are based on international mailing rates, are somewhat higher than domestic rates; please write for details. **THE FIBONACCI QUARTERLY** is published each February, May, August and November.

All back issues of **THE FIBONACCI QUARTERLY** are available in microfilm or hard copy format from **PROQUEST INFORMATION & LEARNING, 300 NORTH ZEEB ROAD, P.O. BOX 1346, ANN ARBOR, MI 48106-1346**. Reprints can also be purchased from **PROQUEST** at the same address.

©2002 by

The Fibonacci Association

All rights reserved, including rights to this journal issue as a whole and, except where otherwise noted, rights to each individual contribution.

# The Fibonacci Quarterly

Founded in 1963 by Verner E. Hoggatt, Jr. (1921-1980)  
and Br. Alfred Brousseau (1907-1988)

THE OFFICIAL JOURNAL OF THE FIBONACCI ASSOCIATION  
DEVOTED TO THE STUDY OF INTEGERS WITH SPECIAL PROPERTIES

## EDITOR

PROFESSOR CURTIS COOPER, Department of Mathematics and Computer Science, Central Missouri State University, Warrensburg, MO 64093-5045 e-mail: cnc8851@cmsu2.cmsu.edu

## EDITORIAL BOARD

DAVID M. BRESSOUD, Macalester College, St. Paul, MN 55105-1899  
JOHN BURKE, Gonzaga University, Spokane, WA 99258-0001  
BART GODDARD, East Texas State University, Commerce, TX 75429-3011  
HENRY W. GOULD, West Virginia University, Morgantown, WV 26506-0001  
HEIKO HARBORTH, Tech. Univ. Carolo Wilhelmina, Braunschweig, Germany  
A.F. HORADAM, University of New England, Armidale, N.S.W. 2351, Australia  
STEVE LIGH, Southeastern Louisiana University, Hammond, LA 70402  
FLORIAN LUCA, Instituto de Matematicas de la UNAM, Morelia, Michoacan, Mexico  
RICHARD MOLLIN, University of Calgary, Calgary T2N 1N4, Alberta, Canada  
GARY L. MULLEN, The Pennsylvania State University, University Park, PA 16802-6401  
HARALD G. NIEDERREITER, National University of Singapore, Singapore 117543, Republic of Singapore  
SAMIH OBAID, San Jose State University, San Jose, CA 95192-0103  
ANDREAS PHILIPPOU, University of Patras, 26100 Patras, Greece  
NEVILLE ROBBINS, San Francisco State University, San Francisco, CA 94132-1722  
DONALD W. ROBINSON, Brigham Young University, Provo, UT 84602-6539  
LAWRENCE SOMER, Catholic University of America, Washington, D.C. 20064-0001  
M.N.S. SWAMY, Concordia University, Montreal H3G 1M8, Quebec, Canada  
ROBERT F. TICHY, Technical University, Graz, Austria  
ANNE LUDINGTON YOUNG, Loyola College in Maryland, Baltimore, MD 21210-2699

## BOARD OF DIRECTORS—THE FIBONACCI ASSOCIATION

FRED T. HOWARD, *President*  
Wake Forest University, Winston-Salem, NC 27109  
PETER G. ANDERSON, *Treasurer*  
Rochester Institute of Technology, Rochester, NY 14623-5608  
GERALD E. BERGUM  
South Dakota State University, Brookings, SD 57007-1596  
KARL DILCHER  
Dalhousie University, Halifax, Nova Scotia, Canada B3H 3J5  
ANDREW GRANVILLE  
University of Georgia, Athens, GA 30601-3024  
HELEN GRUNDMAN  
Bryn Mawr College, Bryn Mawr, PA 19101-2899  
MARJORIE JOHNSON, *Secretary*  
665 Fairlane Avenue, Santa Clara, CA 95051  
CLARK KIMBERLING  
University of Evansville, Evansville, IN 47722-0001  
JEFF LAGARIAS  
AT&T Labs-Research, Florham Park, NJ 07932-0971  
WILLIAM WEBB, *Vice-President*  
Washington State University, Pullman, WA 99164-3113