MATHEMATICAL MODELS FOR THE STUDY OF THE PROPAGATION OF NOVEL SOCIAL BEHAVIOR

HENRY WINTHROP
University of Southern Florida, Tampa, Florida

Suppose we wish to develop a mathematical model for the spread of novel, social behavior, such as rumors, newly coined words, new hobbies or habits, new ideas, etc. Let us illustrate the development of a highly simplified model of this sort, where we are concerned only with behavior which spreads on a person-to-person basis. We shall assume that all individuals who are capable of being potential transmitters of the new behavior adopt it after only one single exposure to it. We shall further assume that all potential transmitters contact exactly m <u>different</u> persons per unit time. Finally, we shall assume a population sufficiently large so that no <u>convergence</u> <u>effects</u> occur during the <u>initial</u> period of growth. By this we mean a population of potential converts whose size, in relation to the actual number of increasing converts, in great enough for <u>practical</u> purposes to warrant the assumption that those who are spreading the novel social behavior will meet for quite some time only individuals who have not as yet been subject to contact with it. This last assumption can be expressed by stating that the rate of repetitious contacts with those who already display the novel behavior in question, is zero.

Under these several constraints it can be shown that the increment of growth, G_i, at any time $t = i$ will be given by

$$G_i = m(m+1)^{i-1}, \quad i \geq 1 \tag{1}$$

and the <u>cumulative</u> or <u>total</u> <u>growth</u>, $N(t)$, in the number of persons who exhibit the novel social behavior at time, t, will be given by

$$N(t) = (m+1)^t, \tag{2}$$

where equation (2) holds only for discrete time instants, that is, where $t = 1, 2, \cdots$.

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We now assume that every person possesses a circle of acquaintances and that, for each person in the population, there are exactly D persons in his circle of acquaintances. We further assume that each person succeeds in contacting all of these D persons only after k units of time have elapsed. In short, $D = mk$. When $t \geq k + 1$ each person continues to exhibit the novel, social behavior but he no longer transmits it to anyone else. G_0 is defined as one. When k units of time have elapsed, the population of converts to the new behavior is $N(k)$. When $t = k + 1$, G_0 will cease to transmit the new behavior but he will still exhibit it. We therefore have

$$N(k + 1) = \left[N(k) - G_0\right] m + N(k) \tag{3}$$

$$= N(k)Y - G_0 m \,, \tag{4}$$

where $Y = (m + 1)$.

At time instant, $t = k + 2$, the number of people who cease to be transmitters will be G_1, and $N(k + 2)$ will be given by the following recursion relationship.

$$N(k + 2) = \left[N(k + 1) - G_1\right] m + N(k + 1) \tag{5}$$

$$= N(k + 1)Y - G_1 m \tag{6}$$

Substituting equation (4) into equation (6) we obtain

$$N(k + 2) = \left[N(k)Y - G_0 m\right] Y - G_1 m \tag{7}$$

which in turn becomes

$$N(k + 2) = N(k)Y^2 - m(G_0 Y + G_1) \tag{8}$$

If we proceed to develop the recursion relationships exhibited in equations (3) through (8), we obtain the following model for $1 \leq i \leq 6$.

$$
\begin{aligned}
N(k+1) &= N(k)Y - G_0 m \\
N(k+2) &= N(k)Y^2 - mY^0(G_0Y + G_1) \\
N(k+3) &= N(k)Y^3 - mY(G_0Y + G_1) - G_2 m \\
N(k+4) &= N(k)Y^4 - mY^2(G_0Y + G_1) - mY^0(G_2Y + G_3) \\
N(k+5) &= N(k)Y^5 - my^3(G_0Y + G_1) - mY(G_2Y + G_3) - G_4 m \\
N(k+6) &= N(k)Y^6 - mY^4(G_0Y + G_1) - mY^2(G_2Y + G_3) - mY^0(G_4Y + G_5) \\
&\vdots
\end{aligned} \tag{9}
$$

From the preceding it can be readily seen that if we wish to determine the value of $N(k + i)$ and if i is even, then

$$
\begin{aligned}
N(k+i) = N(k)Y^i &- mY^{i-2}(G_0Y + G_1) - mY^{i-4}(G_2Y + G_3) - \\
&- mY^{i-6}(G_4Y + G_5) - \cdots - mY^{i-i}(G_{i-2}Y + G_{i-1}),
\end{aligned} \tag{10a}
$$

while if i is odd, then

$$
\begin{aligned}
N(k+i) = N(k)Y^i &- mY^{i-2}(G_0Y + G_1) - mY^{i-4}(G_2Y + G_3) - \cdots \\
&- mY^{i-(i-1)}(G_{i-3}Y + G_{i-2}) - G_{i-1}m
\end{aligned} \tag{10b}
$$

Both equations (10a) and (10b) can be summarized formally as follows.

$$
N(k+i) = N(k)Y^i - m\sum_{n=0}^{i-1} G_n Y^{i-1-n}, \quad 1 \le i \le k \tag{11}
$$

If we substitute $(m + 1)$ for Y into equations (10a) or (10b) and the appropriate value of G_i as given by equation (1), then $N(k + i)$ can be computed. The computed value will reflect the propagation or cumulative growth of the novel social behavior, under all the assumptions and conditions which have been mentioned above.

We now define

$$A \equiv -m\sum_{n=0}^{i-1} G_n Y^{i-1-n} = -mY^{i-1} - m\sum_{n=1}^{i-1} G_n Y^{i-1-n} \tag{12}$$

But by equation (1) we have

$$G_n = m(m+1)^{n-1} = mY^{n-1}, \quad n \geq 1 \tag{13}$$

Hence

$$A = -mY^{i-1} - m^2\sum_{n=1}^{i-1} Y^{i-2} \tag{14}$$

$$= -mY^{i-1} - m^2(i-1)Y^{i-2} \tag{15}$$

If we now substitute the value for A, as given by equation (15), for the second expression on the right-hand side of equation (11), we obtain

$$N(k+i) = N(k)Y^i - mY^{i-1} - m^2(i-1)Y^{i-2}, \quad 1 \leq i \leq k \tag{16}$$

There are two justifications for the constraint that $1 \leq i \leq k$. First is the fact that the growth of the novel behavior will be initially exponential, if the potential population of converts is very much larger than the actual and increasing population of converts for a relatively modest time period occurring at the beginning of the growth phenomenon in question. The actual length of the growth interval assumed is, of course, $2k$ units of time. The second reason for assuming the constraint that $1 \leq i \leq k$ is that the substitution of $i = 0$ in either equations (10a), (10b) or (16), or their analogues, would make no sense. The correction for the fact that transmitters of the novel social behavior possess only a limited circle of acquaintances, D, holds only for those situations in which converted individuals have begun to exhaust their circles of acquaintanceship and, in mathematical terms, this means that $i \neq 0$.

Substituting $(m + 1) = Y$ in equation (16) will yield

$$N(k + i) = (m + 1)^{k+i} - m(m + 1)^{i-1} - (i - 1)m^2(m + 1)^{i-2} \tag{17}$$

$$= (m + 1)^{i-2} (m + 1)^{k+2} - m^2(i - 1) - m(m + 1) \tag{18}$$

$$= (m + 1)^{i-2} (m + 1)^{k+2} - m(im + 1) \tag{19}$$

The equivalence of either equation (10a) with equation (16) or equation (10b) with equation (16), can be seen from the relations given by equations (12) through (15).

The argument of the preceding exposition suggests to some extent how the mathematical model may be of use to the sociologist for a variety of phenomena which are of interest to him.

Models for behavioral diffusion theory have been developed over the last two decades. They may be highly sophisticated or relatively simple, mathematically speaking. Sophisticated examples of models for diffusion theory, intended for some specifically designed experiments, may be found in the work of Rapoport [1]. An early and systematic development of a <u>predominantly</u> algebraic treatment of diffusion theory, intended for experimental designs of an aggregative type, was worked out by Winthrop [2]. The formulation of some early <u>ad hoc</u> models intended for empirical use, was undertaken by Dodd [3]. The relationship of Dodd's S-Theory to those formulations of diffusion theory for which the present writer has been responsible, has been worked out jointly by Dodd and Winthrop [4]. The model presented in this paper is an example of the strictly algebraic type of model. Models of this kind make it somewhat easier to present the exposition of diffusion theory.

REFERENCES

1. A. Rapoport and L. I. Rebhun, "On the Mathematic Theory of Rumor Spread," Bulletion of Mathematic Biophysics, Vol. 14, pp. 375-383, 1952.

2. H. Winthrop, "A Kinetic Theory of Socio-Psychological Diffusion," Journal of Social Psychology, Vol. 22, pp. 31-60, 1945.

3. Stuart C. Dodd, "Testing Message Diffusion from Person to Person," Public Opinion Quarterly, Vol. 16, pp. 247-262, 1952.

4. Stuart C. Dodd and H. Winthrop, "A Dimensional Theory of Social Diffusion," Sociometry, Vol. 16, pp. 180-202, 1953.

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CURIOUS PROPERTY OF ONE FRACTION

J. Wlodarski
Porz-Westhoven, Federal Republic of Germany

It is well known that an integral fraction, with no more than three digits above the line and three below, gives the best possible approximation of the famous mathematical constant "e".

This fraction is 878/323. In decimal form (2,71826 $\cdots$) it yields the correct value for "e" to four decimal places.

If the denominator of this fraction is subtracted from the numerator the difference is 555.

Now, the iterated cross sum of the numerator is 5 and the same cross sum of the denominator is 8. The ratio 5/8 gives the best possible approximation to the "Golden Ratio" with no more than one digit in the numerator and one in the denominator.

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