$$\begin{aligned}(L_{k+2}L_{k+3} - L_kL_{k+1}) &= L_{k+2}(L_{k+2} + L_{k+1}) - L_{k+1}(L_{k+2} - L_{k+1}) \\ &= L_{k+2}^2 + L_{k+1}^2 = (F_{k+3} + F_{k+1})^2 + (F_{k+2} + F_k)^2 \\ &= (F_{k+2} + 2F_{k+1})^2 + (2F_{k+2} - F_{k+1})^2 \\ (3) \qquad &= 5(F_{k+2}^2 + F_{k+1}^2) = 5F_{2k+3} \\ &= 2F_{2k+3} + (F_{2k+5} - F_{2k+4}) + (F_{2k+2} + F_{2k+1}) + F_{2k+3} \\ &= (F_{2k+3} + F_{2k+1}) + (F_{2k+5} + F_{2k+3}) \\ &= L_{2k+2} + L_{2k+4} = C\end{aligned}$$

Thus, from (2) and (3) we have,

$$A^2 + B^2 = C^2 .$$

Also solved by J. A. H. Hunter and A. G. Shannon.

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[Continued from p. 285]

RECURRING SEQUENCES — LESSON 1

ANSWERS TO PROBLEMS

1. $a_n = n(n + 1)$; $T_{n+3} = 3T_{n+2} - 3T_{n+1} + T_n$
2. $a_n = 3n - 2$; $T_{n+2} = 2T_{n+1} - T_n$
3. $a_n = n^3$; $T_{n+4} = 4T_{n+3} - 6T_{n+2} + 4T_{n+1} - T_n$
4. $T_{6n+k} = 1, 3, 3, 1, 1/3, 1/3$, for $k = 1, 2, 3, 4, 5, 6$, respectively
5. $T_{n+1} = \sqrt{1 + T_n^2}$
6. $T_{n+4} = 4T_{n+3} - 6T_{n+2} + 4T_{n+1} - T_n$
7. $T_{n+1} = aT_n$
8. $T_{n+3} = 3T_{n+2} - 3T_{n+1} + T_n$
9. $T_{2n-1} = a$, $T_{2n} = 1/a$
10. $T_{n+1} = 1/(2 - T_n)$

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