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$$|\alpha| > |\beta|, \qquad |\alpha| > 1 ,$$

it is evident that

$$\lim_{N=\infty} \frac{u_{N+n-1}}{u_{N+n}} = \frac{1}{\alpha} .$$

Thus (6.4) implies

$$u_r A_r = u_r \sum_{n=1}^{\infty} \frac{(\alpha\beta)^n}{u_n u_{n+r}} = \sum_{n=1}^{r} \frac{u_{n-1}}{u_n} - \frac{r}{\alpha} . \tag{6.5}$$

Returning to (6.2), we have

$$T_{k+1} = \frac{1}{(u)_{2k}} \sum_{j=0}^{2k} (-1)^j \left\{ {2k \atop j} \right\} (\alpha\beta)^{\frac{1}{2}j(j-1)-jk} \sum_{n=1}^{\infty} \frac{(\alpha\beta)^n}{u_n u_{n+2k-j+1}}$$

$$- \frac{1}{(u)_{2k}} \sum_{j=0}^{2k} (-1)^j \left\{ {2k \atop j} \right\} (\alpha\beta)^{\frac{1}{2}j(j-1)-jk} \sum_{n=1}^{j} \frac{(\alpha\beta)^n}{u_n u_{n+2k-j+1}} .$$

Therefore we have

$$T_{k+1} = \frac{1}{(u)_{2k}} \sum_{j=0}^{2k} (-1)^j \left\{ {2k \atop j} \right\} (\alpha\beta)^{\frac{1}{2}j(j-1)-jk} A_{2k-j+1}$$

$$= \frac{1}{(u)_{2k}} \sum_{j=0}^{2k} (-1)^j \left\{ {2k \atop j} \right\} (\alpha\beta)^{\frac{1}{2}j(j-1)-jk} \sum_{n=1}^{j} \frac{(\alpha\beta)^n}{u_n u_{n+2k-j+1}} .$$

In particular, when $\alpha + \beta = 1$, $\alpha\beta = -1$, (6.6) reduces to

$$\sum_{n=1}^{\infty} \frac{(-1)^{n(k+1)}}{F_n F_{n+1} \cdots F_{n+2k+1}} = \frac{1}{(F)_{2k}} \sum_{j=0}^{2k} (-1)^{\frac{1}{2}j(j+1)-jk} \left\{ \begin{matrix} 2k \\ j \end{matrix} \right\} A_{2k-j+1}$$

$$(6.7) \qquad - \frac{1}{(F)_{2k}} \sum_{j=0}^{2k} (-1)^{\frac{1}{2}j(j+1)-jk} \left\{ \begin{matrix} 2k \\ j \end{matrix} \right\}$$

$$\cdot \sum_{n=1}^{j} \frac{(-1)^n}{F_n F_{n+2k-j+1}} \, ,$$

where now $\left\{ \begin{matrix} 2k \\ j \end{matrix} \right\}$ and $A_{2k-j+1}$ are expressed in terms of Fibonacci numbers.

REFERENCE

1. Brother Alfred Brousseau, "Summation of Infinite Fibonacci Series," Fibonacci Quarterly, Vol. 7 (1969), pp. 143-168.

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$\rho = 1$ stems from its application to the partitioning of integers into distinct Fibonacci numbers. These applications are investigated in the papers listed in References. When $\rho$ is a root of unity, series (1) again has partition — theoretic congruence which we exploited to some extent in Section 5.

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