H. W. Gould
The inverse of a finite series and a third-order recurrent sequence,

Abstract
It is apparently not well-known that \( g(n) = f(n) + f(n-1) + f(n-2) \)
if and only if
\[
 f(n) = \sum_{k=0}^{n} g(n-k) \sum_{j=0}^{\lfloor k/2 \rfloor} (-1)^{k-j} \binom{k-j}{j},
\]
where we suppose that \( f(n) = 0 \) for \( n < 0 \). This may also be expressed
as
\[
 f(n) = \sum_{k=0}^{n} (-1)^k g(n-k) \frac{1}{2} \left( (-1)^{\lfloor k/2 \rfloor} + (-1)^{\lfloor k+1/2 \rfloor} \right).
\]
We show how to solve for \( f(n) \) in the general case
\[
g(n) = \sum_{k=0}^{r} f(n-k), \text{ where } f(n) = 0 \text{ for } n < 0, \text{ with } 1 \leq r \leq n.
\]
We shall also see that the values at which \( g \) is evaluated in forming the
inverse satisfy a third-order recurrence relation of the form
\[
a_n = a_{n-1} + a_{n-2} - a_{n-3}.
\]