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The inverse of a finite series and a third-order recurrent sequence,
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Abstract

It is apparently not well-known that $g(n) = f(n)+f(n-1)+f(n-2)$ if and only if

$$f(n) = \sum_{k=0}^{n} g(n-k) \sum_{j=0}^{[\frac{k}{2}]} (-1)^{k-j} \binom{k-j}{j},$$

where we suppose that $f(n) = 0$ for $n < 0$. This may also be expressed as

$$f(n) = \sum_{k=0}^{n} (-1)^k g(n-k) \frac{1}{2} \left((-1)^{[\frac{k}{3}]} + (-1)^{[\frac{k+1}{3}]} \right).$$

We show how to solve for $f(n)$ in the general case

$$g(n) = \sum_{k=0}^{r} f(n-k), \text{ where } f(n) = 0 \text{ for } n < 0, \text{ with } 1 \le r \le n.$$

We shall also see that the values at which g is evaluated in forming the inverse satisfy a third-order recurrence relation of the form

$$a_n = a_{n-1} + a_{n-2} - a_{n-3}.$$