Wai-Fong Chuan and Fei Yu

*Three new extraction formulae.*

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**Abstract**

Let $\alpha$ be an irrational number between 0 and 1. Let $a$ and $b$ be distinct letters. Define $d_n = a$ (resp., $b$) if $\lfloor (n + 1)\alpha \rfloor - \lfloor n\alpha \rfloor = 0$ (resp., 1), $n \in \mathbb{Z}$. Define $x$ to be the two-way infinite word whose $n$th letter is $d_n, n \in \mathbb{Z}$. Define $x_m = d_{m+1}d_{m+2}\cdots, m \in \mathbb{Z}$, $s_0 = \varepsilon$, the empty word, $s_m = d_1d_2\cdots d_m, m \geq 1$. The problem of determining the extracted word $\langle x_m, x_0 \rangle$ obtained by aligning $x_m$ with $x_0$ was originally posed by D.R. Hofstadter in 1963. Known extraction formulae include $\langle x_m, x_0 \rangle$ ($m > 0$) (by R.J. Hendel and S.A. Monteferrante 1994), $\langle x_0, x_m \rangle$ ($m \geq 1$) (by W. Chuan 1995) for $\alpha = (\sqrt{5} - 1)/2$ and partial results for $\langle x_m, x_0 \rangle$ ($m \geq 1$) (by R.J. Hendel 1996) and all cases of $\langle x_0, x_m \rangle$ ($m \geq 0$) (by W. Chuan and F. Yu 2000) for $\alpha = \sqrt{2} - 1$. In this short note, we establish the following three new extraction formulae for $\alpha = (\sqrt{5} - 1)/2$:

$$
\langle x_m, x_{-2} \rangle = x_m \quad (m > -2)
$$

$$
\langle x_m, x_{-2} \rangle = R(s_{-m-2}) \quad (m \leq -2)
$$

$$
\langle x_0, x_{-m} \rangle = \begin{cases} 
  x_{m-2} & (m > 1) \\
  bx_0 \neq x_{-1} & (m = 1)
\end{cases}
$$

which involve $x_m$, where $m < 0$. We also show that the first formula is equivalent to the formula proved by Hendel and Monteferrante.