The equation \( m^2 - 4k = 5n^2 \) and unique representations of positive integers, Fibonacci Quart. 45 (2007), no. 4, 304–312.

Abstract

If \( n \) is a positive integer, there exists one and only one pair \((j, k)\) of positive integers such that \((j + k + 1)^2 - 4k = 5n^2\). The resulting unique representation of \( n \) can be used to generate both the Wythoff difference array and the Fraenkel array. It also provides the solution of the complementary equation \( b(n) = a(jn) + kn \) in all cases in which \( a \) and \( b \) are a pair of Beatty sequences and \( a(n) \) is of the form \([rn]\) for \( r \) an irrational number in the field \( Q(\sqrt{5}) \).