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The equation $m^2 - 4k = 5n^2$ and unique representations of positive integers, Fibonacci Quart. **45** (2007), no. 4, 304–312.

Abstract

If n is a positive integer, there exists one and only one pair (j, k) of positive integers such that $(j + k + 1)^2 - 4k = 5n^2$. The resulting unique representation of n can be used to generate both the Wythoff difference array and the Fraenkel array. It also provides the solution of the complementary equation $b(n) = a(jn) + kn$ in all cases in which a and b are a pair of Beatty sequences and $a(n)$ is of the form $[rn]$ for r an irrational number in the field $Q(\sqrt{5})$.