Abstract

Let $T : \mathbb{Z} \to \mathbb{Z}$ be defined by $T(x) = \frac{1}{2}x$ if x is even, and $T(x) = \frac{1}{2}(3x+1)$ if x is odd. The $3x+1$ Conjecture asserts that every positive x has an iterate $T^n(x) = 1$. It is known that T^n maps congruence classes with modulus 2^n to those with modulus equal to a power of 3. We describe properties of the image class residues and use those properties to show that, under iteration by T, any congruence class with modulus of the form $2^a 3^b$ generates all integers not divisible by 3. This has negative implications for the study of sufficient sets for the $3x+1$ Conjecture. The analysis also provides insight into a particular permutation function associated with T.