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Some Periodicities in the Continued Fraction Expansions of Fibonacci and Lucas Dirichlet Series,

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Abstract

In this paper we consider the Fibonacci Zeta functions $\zeta_F(s) = \sum_{n=1}^{\infty} F_n^{-s}$ and the Lucas Zeta functions $\zeta_L(s) = \sum_{n=0}^{\infty} L_n^{-s}$. The sequences $\{A_\nu\}_{\nu\geq 0}$ and $\{B_\nu\}_{\nu\geq 0}$, which are derived from $\sum_{\nu=1}^{n} F_\nu^{-s} = A_n/B_n$, satisfy certain recurrence formulas. We examine some properties of the periodicities of A_n and B_n. For example, let m and k be positive integers. If $n \geq mk$, then $B_n \equiv 0 \pmod{F_k^m}$ (with a similar result holding for A_n). The power of 2 which divides B_n is $\left\lfloor \frac{n}{6} \right\rfloor + \sum_{i=0}^{\infty} \left\lfloor \frac{n}{3\cdot 2^i} \right\rfloor$.