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*Some Periodicities in the Continued Fraction Expansions of Fibonacci and Lucas Dirichlet Series*,
Fibonacci Quart. 48 (2010), no. 1, 47–55.

**Abstract**

In this paper we consider the Fibonacci Zeta functions $\zeta_F(s) = \sum_{n=1}^{\infty} F_n^{-s}$ and the Lucas Zeta functions $\zeta_L(s) = \sum_{n=0}^{\infty} L_n^{-s}$. The sequences $\{A_\nu\}_{\nu \geq 0}$ and $\{B_\nu\}_{\nu \geq 0}$, which are derived from $\sum_{\nu=1}^{\infty} F_{\nu}^{-s} = A_{\nu}/B_{\nu}$, satisfy certain recurrence formulas. We examine some properties of the periodicities of $A_n$ and $B_n$. For example, let $m$ and $k$ be positive integers. If $n \geq mk$, then $B_n \equiv 0 \pmod{F_{mk}}$ (with a similar result holding for $A_n$). The power of 2 which divides $B_n$ is $\left\lfloor \frac{n}{6} \right\rfloor + \sum_{i=0}^{\infty} \left\lfloor \frac{n}{2^i} \right\rfloor$. 