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On the Elements of the Continued Fractions of Quadratic Irrationals,
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Abstract

In this paper, we study the elements of the continued fractions of \sqrt{Q} and $(-1 + \sqrt{4Q + 1})/2$ ($Q \in \mathbb{N}$). We prove that if the period length of continued fraction of $(-1 + \sqrt{4Q + 1})/2$ is even, then the middle element is odd (see Theorem 1.4 below), a phenomenon observed first by Arnold [2]. We obtain an analogue theorem for the continued fraction of \sqrt{Q} (see Theorem 1.6 below). We also give the parametrization of positive integers Q such that continued fractions of \sqrt{Q} (respectively, $(-1 + \sqrt{1 + 4Q})/2$) has period of length dividing T , where T is an arbitrary positive integer, which generalize Theorem 3 of Arnold [1]. We explicitly describe the set of positive integers Q such that the continued fraction of \sqrt{Q} has period length equal to 3 or 4.