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Periods of the Tribonacci Sequence Modulo a Prime $p \equiv 1 \pmod{3}$, Fibonacci Quart. 48 (2010), no. 3, 228–235.

Abstract

Let the Tribonacci polynomial $t(x) = x^3 - x^2 - x - 1$ be irreducible over the Galois field $\mathbb{F}_p$ where $p$ is an arbitrary prime such that $p \equiv 1 \pmod{3}$ and let $\tau$ be any root of $t(x)$ in the splitting field $K$ of $t(x)$ over $\mathbb{F}_p$. We prove that $\tau^{(p^2 + p + 1)/3} = 1$. Using this identity we show that the period $h(p)$ of the sequence $(T_n \mod p)_{n=0}^\infty$ where $T_n$ is the $n$th Tribonacci number divides $(p^2 + p + 1)/3$. Similar results will also be obtained for $t(x)$ being reducible over $\mathbb{F}_p$. In this case we prove that the period $h(p)$ divides $(q - 1)/3$ where $q$ is the number of elements of the splitting field of $t(x)$ over $\mathbb{F}_p$ if and only if 2 is a cubic residue of $\mathbb{F}_p$. 