Ralf Bundschuh and Peter Bundschuh
Distribution of Fibonacci and Lucas Numbers Modulo 3^k,
Fibonacci Quart. **49** (2011), no. 3, 201–210.

Abstract

Let $F_0 = 0$, $F_1 = 1$, and $F_n = F_{n-1} + F_{n-2}$ for $n \geq 2$ denote the sequence $\mathcal{F}$ of Fibonacci numbers. For any modulus $m \geq 2$ and residue $b \pmod{m}$, denote by $v_{\mathcal{F}}(m, b)$ the number of occurrences of b as a residue in one (shortest) period of $\mathcal{F}$ modulo m. Moreover, let $v_{\mathcal{L}}(m, b)$ be similarly defined for the Lucas sequence $\mathcal{L}$ satisfying $L_0 = 2$, $L_1 = 1$, and $L_n = L_{n-1} + L_{n-2}$ for $n \geq 2$.

In this paper, completing the recent partial work of Shiu and Chu we entirely describe the functions $v_{\mathcal{F}}(3^k, .)$ and $v_{\mathcal{L}}(3^k, .)$ for every positive integer k. Using a notion formally introduced by Carlip and Jacobson, our main results imply that neither $\mathcal{F}$ nor $\mathcal{L}$ is stable modulo 3. Moreover, in terms of another notion introduced by Somer and Carlip, we observe that $\mathcal{L}$ is a multiple of a translation of $\mathcal{F}$ modulo 3^k (and conversely) for every k.