Abstract

We introduce the function $a(r, n)$ which counts tilings of length $n + r$ that utilize white tiles (whose lengths can vary between 1 and $n$) and $r$ identical red squares. These tilings are called two-toned tilings. We provide combinatorial proofs of several identities satisfied by $a(r, n)$ and its generalizations, including one that produces $k$th order Fibonacci numbers. Applications to integer partitions are also provided.