Lawrence Somer and Michal Křížek
Easy Criteria to Determine if a Prime Divides Certain Second-Order Recurrences,
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Abstract

Let $\mathcal{F}(a, b)$ denote the set of all second-order recurrences $w(a, b)$ satisfying the recursion relation

$$w_{n+2} = aw_{n+1} + bw_n,$$

where the discriminant $D = a^2 + 4b$ and a, b, w_0, and w_1 are all integers. Let $u(a, b)$ denote the recurrence with initial terms $u_0 = 0$ and $u_1 = 1$. We say that the prime p is a divisor of $w(a, b)$ if $p \mid w_n$ for some integer $n \geq 0$. Let $z(p)$ denote the least positive integer n such that $u_n \equiv 0 \pmod{p}$. Then $z(p) \mid p - (D/p)$, where (D/p) denotes the Legendre symbol. Define the index $i(p)$ as

$$i(p) = \frac{p - (D/p)}{z(p)}.$$

When $i(p) = 1$ or 2, we will find easy criteria to determine exactly when p is a divisor of $w(a, b)$ based on the residue class or quadratic character of $w_1^2 - aw_1w_0 - bw_0^2$ modulo p. This generalizes results of Vandervelde when $a = b = 1$.