Abstract

An $n$-color composition of $n$ is a composition of $n$ where a part $k$ has $k$ possible colors. It is known that the number of $n$-color compositions of $n$ is $F_{2n}$ (the $2n$th Fibonacci numbers). Among other objects, $F_{2n}$ also counts the number of binary words with exactly $n - 1$ strictly increasing runs and the number of \{0, 1, 2\} strings of length $n - 1$ excluding the subword 12. In this note, we show bijections between $n$-color compositions and these objects. In particular, the bijection between the $n$-color compositions and the binary words with $n - 1$ increasing substrings generalizes the classic bijection between compositions and binary words of length $n - 1$. We also comment on the potential applications of these findings.