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*Sharper Upper Bounds for the Order of Appearance in the Fibonacci Sequence,*

**Abstract**

Let $F_n$ be the $n$th Fibonacci number. The order of appearance $z(n)$ of a natural number $n$ is defined as the smallest natural number $k$ such that $n$ divides $F_k$. In 1975, J. Sallé proved that $z(n) \leq 2n$, for all positive integers $n$. In this paper, we shall provide sharper upper bounds for $z(n)$ which are substantially smaller than $2n$ for some values of $n$. Moreover, we shall prove that

$$\liminf_{n \to \infty} \frac{z(n)}{n} = 0.$$