Ana Paula Chaves and Diego Marques
A Diophantine Equation Related to the Sum of Squares of Consecutive k-Generalized Fibonacci Numbers,
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Abstract

Let $(F_n)_{n\geq 0}$ be the Fibonacci sequence given by $F_{n+2} = F_{n+1}+F_n$, for $n \geq 0$, where $F_0 = 0$ and $F_1 = 1$. There are several interesting identities involving this sequence such as $F_n^2 + F_{n+1}^2 = F_{2n+1}$, for all $n \geq 0$. One of the most known generalizations of the Fibonacci sequence, is the k-generalized Fibonacci sequence $(F_n^{(k)})_n$ which is defined by the initial values $0, 0, \ldots, 0, 1$ (k terms) and such that each term afterwards is the sum of the k preceding terms. In this paper, we prove that contrarily to the Fibonacci case, the Diophantine equation

$$(F_n^{(k)})^2 + (F_{n+1}^{(k)})^2 = F_m^{(k)}$$

has no any solution in positive integers n, m and k, with $n > 1$ and $k \geq 3$.