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Abstract

We evaluate two continued fractions whose elements contain Fibonacci numbers indexed by the Fibonacci and Lucas sequences. One of the results obtained is

$$\frac{1+\sqrt{5}}{2} = 1 + \frac{F_{F_1}/F_{F_4}}{1} - \frac{F_{F_2}/F_{F_5}}{1} + \frac{F_{F_3}/F_{F_6}}{1} + \frac{F_{F_4}/F_{F_7}}{1} - \frac{F_{F_5}/F_{F_8}}{1} + \frac{F_{F_6}/F_{F_9}}{1} + \cdots .$$

Similar results with other rapidly growing sequences of subscripts are provided and associated summation theorems are also given. These results are shown to fit naturally in the context of a general transformation formula for arbitrary continued fractions due to Oskar Perron.