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Symmetries of Fibonacci Points, Mod m,

Abstract

Given a modulus \( m \), we examine the set of all points \((F_i, F_{i+1}) \in \mathbb{Z}_m^2\) where \( F \) is the usual Fibonacci sequence. We graph the set in the fundamental domain \([0, m - 1] \times [0, m - 1]\), and observe that as \( m \) varies, sometimes the graph appears as a random scattering of points, but often it shows striking symmetry. We prove that in exactly three cases \((m = 2, 3, \text{ or } 6)\) the graph shows symmetry by reflection across the line \( y = x \). We prove that symmetry by rotation occurs exactly when the terms 0, −1 appear half-way through a period of \( F \) (mod \( m \)). We prove that symmetry by translation can occur in essentially one way, and we provide conditions equivalent to the graph having symmetry by translation.