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Symmetries of Fibonacci Points, Mod m ,
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Abstract

Given a modulus m , we examine the set of all points $(F_i, F_{i+1}) \in \mathbb{Z}_m^2$ where F is the usual Fibonacci sequence. We graph the set in the fundamental domain $[0, m - 1] \times [0, m - 1]$, and observe that as m varies, sometimes the graph appears as a random scattering of points, but often it shows striking symmetry. We prove that in exactly three cases ($m = 2, 3, \text{ or } 6$) the graph shows symmetry by reflection across the line $y = x$. We prove that symmetry by rotation occurs exactly when the terms $0, -1$ appear half-way through a period of $F \pmod{m}$. We prove that symmetry by translation can occur in essentially one way, and we provide conditions equivalent to the graph having symmetry by translation.