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*Congruences for Partial Sums of Reciprocals,*


**Abstract**

We systematically derive congruences for the sums \( \sum_{j=1}^{\lfloor kp/M \rfloor} 1/j^2 \) modulo \( p \) and for the sums \( \sum_{j=1}^{\lfloor kp/M \rfloor} 1/j \) modulo \( p^2 \), for all integers \( M \geq 1 \) that divide 24 and integers \( k \) with \( 1 \leq k \leq M \) and \( \gcd(M, k) = 1 \). While many of these congruences are well-known, others are new in the forms given. The congruences involve Fermat quotients, Euler numbers, Bernoulli polynomials, and some particular classes of generalized Bernoulli numbers belonging to quadratic characters.