Abstract

Let \( w(a, -1) \) denote the second-order linear recurrence satisfying the recursion relation

\[
w_{n+2} = aw_{n+1} - w_n,
\]

where \( a \) and the initial terms \( w_0, w_1 \) are all integers. Let \( p \) be an odd prime. The restricted period \( h_w(p) \) of \( w(a, -1) \) modulo \( p \) is the least positive integer \( r \) such that \( w_{n+r} \equiv Mw_n \pmod{p} \) for all \( n \geq 0 \) and some nonzero residue \( M \) modulo \( p \). We distinguish two recurrences, the Lucas sequence of the first kind \( u(a, -1) \) and the Lucas sequence of the second kind \( v(a, -1) \), satisfying the above recursion relation and having initial terms \( u_0 = 0, u_1 = 1 \) and \( v_0 = 2, v_1 = a \), respectively. We show that if \( u(a_1, -1) \) and \( u(a_2, -1) \) both have the same restricted period modulo \( p \), or equivalently, the same period modulo \( p \), then \( u(a_1, -1) \) and \( u(a_2, -1) \) have the same distribution of residues modulo \( p \). Similar results are obtained for Lucas sequences of the second kind.