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A Problem on Generation Sets Containing Fibonacci Numbers,
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Abstract

At the Sixteenth International Conference on Fibonacci Numbers and Their Applications the following problem was posed by Clark Kimberling:

Let S be the set generated by these rules: Let $1 \in S$ and if $x \in S$, then $2x \in S$ and $1 - x \in S$, so that S grows in generations:

$$G_1 = \{1\}, G_2 = \{0, 2\}, G_3 = \{-1, 4\}, \ldots.$$

Prove or disprove that each generation contains at least one Fibonacci number or its negative.

In this paper we generalize the problem as follows. Let S be the set described above, $\mathcal{S}$ be a sequence and $\mathcal{P}$ the property that a generation contains a term of $\mathcal{S}$ or the negative of a term of $\mathcal{S}$. We will show that when $\mathcal{S}$ is the Fibonacci sequence there are many generations that fail to have property $\mathcal{P}$. Other sequences $\mathcal{S}$ will also be considered and shown to have at least one generation failing to have property $\mathcal{P}$. The proportion of generations failing to have property $\mathcal{P}$ is also investigated.