Abstract

Let $K/Q$ be a finite Galois extension. A normal integral basis for $K$ is an integral basis for $K$ in which all the elements of the basis are conjugate over $Q$. Let $\theta \in \mathbb{R}$ be a root of the polynomial

$$f(X) = X^5 + X^4 - 4X^3 - 3X^2 + 3X + 1.$$ 

Set $K = \mathbb{Q}(\theta)$. It is known that $K$ possesses infinitely many normal integral bases. In this paper, we explicitly determine all normal integral bases of $K$ and parametrize them using the Fibonacci and Lucas numbers.