Abstract

In this paper, we present closed forms for certain finite sums of weighted products of generalized Fibonacci numbers. Indeed, we present seven multi-parameter families of such finite sums, all of which we believe to be new. In each of these families, the number of factors in the summand is governed by the size of the integer parameter \( j \geq 1 \), and can be made as large as we please.

We present our main results in terms of sequences that generalize the Fibonacci/Lucas numbers. Consequently, each of our main results can be specialized to involve the Fibonacci/Lucas numbers. For instance, as a consequence of one of our main results, it follows that

\[
\sum_{i=1}^{n} 2^{i-1} F_i F_{i+2} = 2^n F_n F_{n+1}.
\]

Here the weight term in the summand is \( 2^{i-1} \).