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What I Tell You K Times is True ...,
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Abstract

Some years ago while investigating generalized Zeckendorf representations, i.e. representations of integers as binary sums of k^{th} order Fibona cci numbers, we found that the fraction of 0's in the representation was a monotone increasing function of the number of bits used. It was relatively easy to show that this behavior was to be expected asymptotically, but was there an easy way to show that this fraction was always increasing? Specifically, could one find a K (presumably bigger than the order k) so that if the fraction was increasing for K consecutive steps, then it would always be increasing? Here, we introduce the **SP** (sorta positive) polynomials. We show that if th e characteristic polynomial for a difference equation is a factor of an **SP** p olynomial of degree K, then if the ratios of any two solutions to the equation are K in row increasing, then the ratios are always increasing. For the k^{th} order Fibonacci sequences $K = k + 1$. For the fraction of 0's in the Zeckendorf representation, the characteristic polynomial is the square of a Fibonacci characteristic polynomial (i.e. degree $= 2k$) and we show that $K = 2k + 2$.