John Greene

*Lucas Sequences and Traces of Matrix Products*,

Fibonacci Quart. **56** (2018), no. 3, 200–211.

**Abstract**

Given two noncommuting matrices, $A$ and $B$, it is well-known that $AB$ and $BA$ have the same trace. This extends to cyclic permutations of products of $A$’s and $B$’s. Thus if $A$ and $B$ are fixed matrices, then products of two $A$’s and four $B$’s can have three possible traces. For $2 \times 2$ matrices $A$ and $B$, we show that there are restrictions on the relative sizes of these traces. For example, if $M_1 = AB^2AB^2$, $M_2 = ABAB^3$, and $M_3 = A^2B^4$, then it is never the case that $\text{Tr}(M_2) > \text{Tr}(M_3) > \text{Tr}(M_1)$, but the other five orderings of the traces can occur. By utilizing the connection between Lucas sequences and powers of a $2 \times 2$ matrix, a formula is given for the number of orderings of the traces that can occur in products of two $A$’s and $n$ $B$’s.