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Lucas Sequences and Traces of Matrix Products,

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Abstract

Given two noncommuting matrices, A and B, it is well-known that AB and BA have the same trace. This extends to cyclic permutations of products of A's and B's. Thus if A and B are fixed matrices, then products of two A's and four B's can have three possible traces. For 2×2 matrices A and B, we show that there are restrictions on the relative sizes of these traces. For example, if $M_1 = AB^2AB^2$, $M_2 = ABAB^3$, and $M_3 = A^2B^4$, then it is never the case that $\mathrm{Tr}(M_2) > \mathrm{Tr}(M_3) > \mathrm{Tr}(M_1)$, but the other five orderings of the traces can occur. By utilizing the connection between Lucas sequences and powers of a 2×2 matrix, a formula is given for the number of orderings of the traces that can occur in products of two A's and n B's.