Helen G. Grundman and Pamela E. Harris
Sequences of Consecutive Happy Numbers in Negative Bases,
Fibonacci Quart. **56** (2018), no. 3, 221–228.

Abstract

For $b \leq -2$ and $e \geq 2$, let $S_{e,b} : \mathbb{Z} \to \mathbb{Z}_{\geq 0}$ be the function taking an integer to the sum of the e-powers of the digits of its base b expansion. An integer a is a *b-happy number* if there exists $k \in \mathbb{Z}^+$ such that $S_{2,b}^k(a) = 1$. We prove that an integer is -2-happy if and only if it is congruent to 1 modulo 3 and that it is -3-happy if and only if it is odd. Defining a d-sequence to be an arithmetic sequence with constant difference d and setting $d = \gcd(2, b-1)$, we prove that for odd $b \leq -3$ and for $b \in \{-4, -6, -8, -10\}$, there exist arbitrarily long finite sequences of d-consecutive b-happy numbers.