Explicit Bounds for the Diophantine Equation $A!B! = C!$,

Abstract

A nontrivial solution of the equation $A!B! = C!$ is a triple of positive integers $(A, B, C)$ with $A \leq B \leq C - 2$. It is conjectured that the only nontrivial solution is $(6, 7, 10)$, and this conjecture has been checked up to $C = 10^6$. Several estimates on the relative size of the parameters are known, such as the one given by Erdős, $C - B \leq 5 \log \log C$, or the one given by Bhat and Ramachandra, $C - B \leq (1/\log 2 + o(1)) \log \log C$. We check the conjecture for $B \leq 10^{3000}$ and give better explicit bounds such as $C - B \leq \frac{\log \log(B+1)}{\log 2} - 0.8803$. 