Neelima Borade, Dexter Cai, David Z. Chang, Bruce Fang, Alex Liang, Steven J. Miller, and Wanqiao Xu
Gaps of Summands of the Zeckendorf Lattice,
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Abstract

A theorem of Zeckendorf states that every positive integer has a unique decomposition as a sum of nonadjacent Fibonacci numbers. Such decompositions exist more generally, and much is known about them. First, for any positive linear recurrence $\{G_n\}$, the number of summands in the legal decompositions for integers in $[G_n, G_{n+1})$ converges to a Gaussian distribution. Second, Bower, Insoft, Li, Miller, and Tosteson proved that in a legal decomposition, the probability of a gap between summands, that is larger than the recurrence length, converges to geometric decay. Whereas most of the literature involves one-dimensional sequences, some recent work by Chen, Guo, Jiang, Miller, Siktar, and Yu have extended these decompositions to d-dimensional lattices, where a legal decomposition is a chain of points such that one moves in all d dimensions to get from one point to the next. They proved that some but not all properties from one-dimensional sequences still hold. We continue this work and look at the distribution of gaps between terms of legal decompositions, and prove, similar to the one-dimensional cases, that the gap vectors converge to a bivariate geometric random variable when $d = 2$.