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A New Combinatorial Interpretation of the Fibonacci Numbers Squared. Part II.

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Abstract

We give further combinatorial proofs of identities related to the Fibonacci numbers squared by considering the tiling of an n-board (a $1 \times n$ array of square cells of unit width) with half-squares ($\frac{1}{2} \times 1$ tiles) and $(\frac{1}{2}, \frac{1}{2})$-fence tiles. A (w, g)-fence tile is composed of two $w \times 1$ rectangular subtiles separated by a gap of width g. In addition, we construct a Pascal-like triangle whose (n, k)th entry is the number of tilings of an n-board that contain k fences. Elementary combinatorial proofs are given for some properties of the triangle and we show that reversing the rows gives the $(1/(1-x^2), x/(1-x)^2)$ Riordan array. Finally, we show that tiling an n-board with $(\frac{1}{4}, \frac{1}{4})$- and $(\frac{1}{4}, \frac{3}{4})$-fences also generates the Fibonacci numbers squared.