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A New Combinatorial Interpretation of the Fibonacci Numbers Squared.

Part II.


Abstract

We give further combinatorial proofs of identities related to the Fibonacci numbers squared by considering the tiling of an \( n \)-board (a \( 1 \times n \) array of square cells of unit width) with half-squares (\( \frac{1}{2} \times 1 \) tiles) and (\( \frac{1}{2}, \frac{1}{2} \))-fence tiles. A \((w, g)\)-fence tile is composed of two \( w \times 1 \) rectangular subtiles separated by a gap of width \( g \). In addition, we construct a Pascal-like triangle whose \((n, k)\)th entry is the number of tilings of an \( n \)-board that contain \( k \) fences. Elementary combinatorial proofs are given for some properties of the triangle and we show that reversing the rows gives the (\( \frac{1}{1-x^2}, \frac{x}{1-x^2} \)) Riordan array. Finally, we show that tiling an \( n \)-board with (\( \frac{1}{4}, \frac{1}{4} \))- and (\( \frac{1}{4}, \frac{3}{4} \))-fences also generates the Fibonacci numbers squared.