Yuji Tsuno
Extended Results on Integer Values of the Generating Functions for Sequences Given by Pell's Equations,
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Abstract

Hong posed the question when rational numbers map to integers for the generating function of Fibonacci numbers. This problem was solved by Pongsriiam and independently by Bulawa and Lee. The key to solving this problem is to consider the Fibonacci sequence and the Lucas sequence as sequences obtained from the integer solutions of Pell's equation $5x^2-y^2 = \pm 4$. In this study, based on previous research, we change Hong's question and consider the case of the generating functions for the sequences obtained from the integer solutions of Pell's equation $x^2 - dy^2 = \pm 1$ (d is a nonsquare natural number). Similar to previous results, our main results are expressed in the form of ratios of adjacent terms of the sequences obtained from the integer solutions of Pell's equation $x^2 - dy^2 = \pm 1$. Furthermore, the results of Bulawa and Lee pertained to a class of sequences with recurrence relations that were more generalized than those obeyed by the Fibonacci and Lucas sequences. These sequences in our study arise as solutions to the equation $x^2 - dy^2 = \pm 1$, and, as such, obey the type of recurrence relations considered by Bulawa and Lee; however, the initial conditions of these sequences were not considered by those authors. Therefore, our study extends previous research.