

Patrick Moss and Tom Ward
Fibonacci Along Even Powers Is (Almost) Realizable,
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Abstract

An integer sequence is called realizable if it is the count of periodic points of some map. The Fibonacci sequence (F_n) does not have this property, and the Fibonacci sequence sampled along the squares (F_{n^2}) also does not have this property. We show that the former is an irreparable feature of the Fibonacci sequence, whereas the latter is an arithmetic phenomenon related to the discriminant of the Fibonacci sequence by showing that (F_n) fails a congruence condition at infinitely many primes, whereas the sequence $(5F_{n^2})$ is realizable. More generally, we show that $(F_{n^{2k-1}})$ is not realizable in a particularly strong sense, whereas $(5F_{n^{2k}})$ is realizable, for any $k \geq 1$.