PREFACE

The discrete methods of combinatorial analysis, and their application to the construction of mathematical models and solutions of applied problems in technology and the natural sciences, have brought about a great deal of interest in the study of the arithmetic and geometric properties of the so-called "arithmetic triangles." The classical example of the arithmetic triangle is, of course, the Pascal triangle.

In recent decades there has been a widening circle of research on the Pascal triangle itself, as well as its planar and spatial analogs and generalizations. Although there are a large number of scientific and methodological papers devoted to the study of the Pascal triangle and other arithmetic triangles, there have been only a few isolated expository studies and books, chiefly methodological in character. Among these we might cite the small volume of V.A. Uspenskii "The Pascal Triangle" [50], which was translated into English and gives a popular account of the basic properties of the Pascal triangle, and also the excellent book of T.M. Green and C.L. Hamberg, "Pascal's Triangle " [162], which describes known and new properties of the Pascal triangle, and is intended for college students and amateur mathematicians.

The present monograph is devoted to rather more profound questions connected with the study of the Pascal triangle, and its planar and spatial analogs. There is an extensive discussion of the divisibility of the binomial, trinomial, and multinomial coefficients by a prime p, and of the distributions of these coefficients with respect to the modulus p, or p^s, in 8? corresponding arithmetic triangles, pyramids, and hyperpyramids. Particular attention is given to those objects which today we speak of as fractals, and whose present extensive development arose from the works of Benoit Mandelbrot [270-272]. Fractals obtained from the Pascal triangle and other arithmetic triangles are described, as are also results from the study of the properties of the generalized arithmetic graph, a special case of which is the graph model of the generalized Pascal triangle. We also construct and investigate matrices and determinants whose elements may be binomial, generalized binomial, and trinomial coefficients, and other special values. Particular attention is given to the development of effective combinatorial methods and algorithms for the construction of basis systems of polynomial solutions of partial differential equations, including equations of high order and with mixed derivatives. The algorithms proposed are invariant with respect to the order, and the iteration, of operators arising in connection with differential equations. Finally, we discuss non-orthogonal polynomials of binomial type, and polynomials whose coefficients may be Fibonacci, Lucas, Catalan, and other special numbers.

This monograph consists of seven chapters, and there are many illustrations and specific examples. Fundamental results are formulated as theorems and algorithms, and as various equations and formulas. There is a detailed list of over four hundred references, covering almost all known works on arithmetic triangles and pyramids,

The author is deeply grateful to S.G. Mikhlin for his valuable advice and constant support of this work, and to A.A. Adylov for writing Chapter 5 on arithmetic graphs.

For their reviews of the manuscript and useful comments the author thanks F.B. Abutaliev, V.M. Maksimov, and V.K. Kabulov.

The solutions of the specific examples, and the constructions of the arithmetic triangles and their fractals, were carried out by Mariya Morozova, to whom the author gladly expresses his appreciation.

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