$$\sum_{i \geq 0} \binom{n}{i} 5^{\lfloor i/2 \rfloor} = 2^n F_{n+1}$$

Even terms

Odd terms

$$\sum_{i \geq 0} \binom{n}{2i} 5^i = 2^{n-1} L_n$$

$$\sum_{i \geq 0} \binom{n}{2i+1} 5^i = 2^{n-1} F_n$$