"FIBONACCI'S LIBER ABACI": A TRANSLATION INTO MODERN ENGLISH OF LEONARDO PISANO'S BOOK OF CALCULATION, BY L.E. SIGLER

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REVIEW

Fibonacci's Liber Abaci: A Translation into Modern English of Leonardo Pisano's Book of Calculation, by L.E. Sigler [Springer 2002].

BACKGROUND

"The nine Indian figures are:

9 8 7 6 5 4 3 2 1.

With these nine figures and with sign 0 which the Arabs call *zephir* any number whatsoever is written, as is demonstrated below."

Thus begins the book by Fibonacci (Leonardo Pisano, family Bonaci) which many mathematicians have waited for a long time for some gifted translator to produce.

Appearing first in 1202 (i.e., exactly eight centuries before this translation), and again in 1228 in another version, the monumental and comprehensive *Liber abaci* is one of the seminal contributions in mathematical history, and not merely in medieval mathematics.

This is the first translation into a modern language of the Latin manuscript of Fibonacci's original and striking creative work. It is based on the "complete and unambiguous" (p. 11) printed edition by Boncompagni [1] in Italy in 1857. This definitive text is itself founded on numerous manuscripts in Europe of *Liber abaci* studied by Boncompagni. (Paradoxically, *abaco* is the process of calculation with the Hindu-Arabic numbers **without** using the calculational powers of the abacus.)

An excellent Introduction (pp. 1-11) provides an overview of Fibonacci's life and great achievements. Valuable explanatory Notes (pp. 617-633) at the end of the translation add to the enjoyment and depth of understanding of the text. Appended within the Notes is a selection of Fibonacci's problems solved by means of modern algebraic symbolism.

Readers may wish to consult related Fibonacci material in [3], [4], and [5].

IMPORTANCE

The significance of *Liber abaci* lies in its dissemination throughout Europe, and later the world, of the Hindu ("Indian") - Arabic numeration system, in which 0 is a place holder, and of the geometrical and latent algebraic knowledge displayed in Fibonacci's methods. The validity of his treatment is justified by proofs based on the principles applied in Euclidean mathematics, "the subtle Euclidean geometric art" (p. 16). Use of the advanced Hindu-Arabic system of numerals, gained through Fibonacci's commercial connections in North Africa and the Levant, had an enormous advantage over the cumbersome Roman numerals. It must be remembered that Fibonacci's home city-state of Pisa had an extensive mercantile fleet operating in, and beyond, the Mediterranean to Byzantium.

Emphasis should be given to some concepts and usages in *Liber abaci* not widely viewed as existing in the mathematical knowledge of his day, namely,

(i) Fibonacci's familiarity with ideas equivalent to algebraic symbolism,
(ii) his understanding of negative numbers, and
(iii) his use of fractions capable of representing our decimal system.

Elementary modular arithmetic and Diophantine thinking were available to him as tools in problem-solving.

NATURE OF THE CONTENTS

After explaining in detail the virtues of the revolutionary new Hindu-Arabic system, Fibonacci proceeds to set out with meticulous thoroughness the operations for addition, subtraction, multiplication, and division, always stressing the need for practice in calculating with these numbers.

Theoretic work on integers is extended to manipulation of fractions, but more of this anon. Solutions to many problems involve the use of proportions and an elementary understanding of equations. In general, the mathematics of *Liber abaci* encompasses much knowledge of previous discoveries including those of Pythagoras, Euclid, and Diophantus, together with the algorithmic methods derived from Arab sources, such as al-Khowārismī, and others. All this is capped by Fibonacci's particular creative intelligence and organisational prowess in assembling his material. Indeed, *Liber abaci* is a veritable compendium of medieval mathematics and thus a source of valuable knowledge for succeeding generations of scholars.

How is this wide-ranging information applied? Bearing in mind Fibonacci's commercial background in the Mediterranean world, we should not be surprised that applications of his mathematics extended, *inter alia*, to: weights and measures (from which we learn a lot about medieval monetary transactions), barter systems, business accounting, company profits and investments, and alloying of coins, along with general problems relating to the purchase and sale of animals, birds, and grain. Mixed with these matters of ordinary business and trading affairs there occur many theoretical problems, all of which powerfully display the versatility, depth of comprehension, and agility of Fibonacci's mind.

A Representative Problem: Alloying of coins (silver and copper) (pp. 234-235, p. 623).

"If you will have two major monies of which one is with 7 ounces and the other with 6 ounces from which you wish to make one pound of money in which there are 4 ounces of silver, then you will wish to know how many ounces of each money, and how much copper are to be adjoined."

[A Pisan ounce (*uncia*) = $\frac{1}{12}$ Pisan pound.]

As Fibonacci acknowledges, this Diophantine-type problem has many solutions such as putting in, from all three of the posed monies, amounts which are (i) equal, (ii) unequal, and (iii) proportional.

FRACTIONS

¿From historical readings we know the degree to which manipulations with fractions often perplexed some of the acutest intellects of their times. Fibonacci adapted a flexible, but compact, notation for handling this testing aspect of arithmetic. Representative illustrations of his usage of fractional notation and processes occur on pages 49 *et seq.* of the text and on p. 619 in the translator's Notes, from which these examples are extracted:

$$\frac{14}{27}=\frac{1}{2\times 7}+\frac{4}{7}\left(=\frac{9}{14}\right),\ \frac{10}{27}=\frac{1}{2\times 7}+\frac{0}{7}\left(=\frac{1}{14}\right),$$

$$\frac{157}{2610}=\frac{1}{2\times 6\times 10}+\frac{5}{6\times 10}+\frac{7}{10}\left(=\frac{13}{24}\right),$$

$$\frac{2468}{3579}\circ=\frac{2\times 4\times 6\times 8}{3\times 5\times 7\times 9}+\frac{4\times 6\times 8}{5\times 7\times 9}+\frac{6\times 8}{7\times 9}+\frac{8}{9}\left(=\frac{8}{3}\right),$$

$$\circ\frac{8642}{9753}=\frac{8\times 6\times 4\times 2}{9\times 7\times 5\times 3}\left(=\frac{128}{315}\right),$$

$$\frac{\underline{1}\;\underline{1}\;\underline{1}\;5}{5439}=\frac{1}{5\times 9}+\frac{1}{4\times 9}+\frac{1}{3\times 9}+\frac{5}{9}\left(=\frac{347}{540}\right),$$

in which the small symbol $\circ$ ("a circle") does not represent zephir. Fibonacci also reveals a predilection for the Egyptian method of expressing a fraction as a sum of fractions with unit numerators ("unit fractions").

Such variety and generality of Fibonacci's usage is needed to deal with the many different measures and currencies encountered by him.

Though the Pisan currency was not decimal, Fibonacci employs the decimal ideal when required. For example, on p. 439, he poses the problem of a man with 100 bezants (1 bezant = 10 mils) who travelled through 12 cities and had to give in each of these cities one tenth of his bezants. How many bezants remain after leaving the 12th city?

Fibonacci gives the amount of bezants spent as

$$\frac{1\quad 8\quad 4\quad 6\quad 3\quad 5\quad 9\quad 2\quad 4\quad 2}{10\ \ 10\ \ 10\ \ 10\ \ 10\ \ 10\ \ 10\ \ 10\ \ 10\ \ 10}28(=28.2429536481)$$

the whole number being written to the right of the fractional component. Clearly the amount remaining is the original 100 bezants minus this quantity of bezants.

So huge is Fibonacci's treatise on fractions that no real justice can be done to it in a restricted space.

Further information on Fibonacci's Egyptian fractions may be gleaned from [2], but note the distinctive rules (Notes p. 620) employed by the authors of [2] for restating Fibonacci's composite fractions in our ordinary fractional rotation.

SUMMARY

While this reviewer does not consider himself competent to comment on the accuracy of the skills of the translator, it is enough to assert that so vast an undertaking reads uniformly smoothly, a worthy tribute to Sigler's expertise. This review is only intended to give a broad picture of, and a feeling for, the main accomplishments of Fibonacci and Sigler.

For any dereliction of duty, I take refuge in Fibonacci's modest statement (p. 16) in his Dedication and Prologue:

"If, by chance, something less or more proper or necessary I omitted, your indulgence for me is entreated, as there is no one who is without fault, and in all things altogether circumspect."

REFERENCES

[1] B. Boncompagni, *Scritti di Leonardo Pisano*, Rome 1857.
[2] M. Dunton & R.E. Grimm. "Fibonacci on Egyptian Fractions." *The Fibonacci Quarterly* **4.4** (1966): 339-354.
[3] R.E. Grimm. "The Autobiography of Leonardo Pisano." *The Fibonacci Quarterly* **11.1** (1973): 99-104.
[4] A.F. Horadam. "Fibonacci's Mathematical Letter to Master Theodorus." *The Fibonacci Quarterly* **29.2** (1991): 103-107.
[5] L.E. Sigler. "The Book of Squares." Translation of Fibonacci's *Liber Quadratorum.* Reviewed by A.F. Horadam, *The Fibonacci Quarterly* **26.4** (1988): 382.

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