CORRECTIONS TO "CONNECTION COEFFICIENTS FOR HIGHER-ORDER BERNOULLI AND EULER POLYNOMIALS: A RANDOM WALK APPROACH"

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We apologize for a mistake in Theorem 4.5 on page 94 [1]. The correct statement and proof should be as follows.

Theorem 4.5 For any positive integer n, we have

$$\frac{B_{n+1}\left(\frac{x+4}{8}\right)-B_{n+1}\left(\frac{x}{8}\right)}{n+1}=\frac{1}{2^{2n+1}}\sum_{k\geq 0}\frac{1}{2^{k+1}}E_n^{(2k+2)}\left(\frac{x}{2}+k\right).$$

Proof. From

$$\phi_{0\to 4}(z)=\phi_{0\to 1}(z)\phi_{1\to 2|\not{0}}(z)\phi_{2\to 3|\not{1}}(z)\phi_{3\to 4|\not{2}}(z)\sum_{k\geq 0}\left(I_{1,2}+I_{2,3}\right)^k,$$

namely,

$$\frac{4w}{\sinh(4w)}=\frac{w}{\sinh w}\cdot\frac{2\sinh w}{\sinh(2w)}\cdot\frac{3\sinh w}{2\sinh(2w)}\cdot\frac{4\sinh w}{3\sinh(2w)}\sum_{k\geq 0}\frac{\operatorname{sech}^{2k}(w)}{2^k}=\frac{w}{\sinh(2w)}\sum_{k\geq 0}\frac{\operatorname{sech}^{2k+2}(w)}{2^k},$$

we deduce that

$$e^{4w(2\mathcal{B}+1)}=\frac{1}{2}e^{2w(2\mathcal{B}'+1)}\sum_{k\geq 0}\frac{1}{2^k}e^{w\left(2\mathcal{E}^{(2k+2)}+2k+2\right)}.$$

After multiplying by e^{xw}, applying the change of variable $x\mapsto x-4$, and identifying the coefficients of w^n on both sides, we have

$$(x+8\mathcal{B})^n=\sum_{k\geq 0}\frac{\left(x+4\mathcal{B}'+2\mathcal{E}^{(2k+2)}+2k\right)^n}{2^{k+1}}.$$

Now we apply the substitution $x\mapsto x+4\mathcal{U}$ to obtain for the left-hand side,

$$\begin{aligned}(x+8\mathcal{B}+4\mathcal{U})^n&=\int_0^1(x+8\mathcal{B}+4u)^n\,du\\&=\frac{(x+8\mathcal{B}+4)^{n+1}-(x+8\mathcal{B})^{n+1}}{4(n+1)}\\&=\frac{8^{n+1}}{4(n+1)}\left(\left(\mathcal{B}+\frac{x+4}{8}\right)^{n+1}-\left(\mathcal{B}+\frac{x}{8}\right)^{n+1}\right)\\&=\frac{2^{3n+1}\left(B_{n+1}\left(\frac{x+4}{8}\right)-B_{n+1}\left(\frac{x}{8}\right)\right)}{n+1},\end{aligned}$$

while for the right-hand side,

$$\sum_{k\geq 0}\frac{\left(x+2\mathcal{E}^{(2k+2)}+2k\right)^n}{2^{k+1}}=2^n\sum_{k\geq 0}\frac{\left(\mathcal{E}^{(2k+2)}+\frac{x+2k}{2}\right)^n}{2^{k+1}}=2^n\sum_{k\geq 0}\frac{E_n^{(2k+2)}\left(\frac{x}{2}+k\right)}{2^{k+1}}.$$

Further simplification completes the proof. □

References

[1] L. Jiu and C. Vignat, Connection coefficients for higher-order Bernoulli and Euler polynomials: a random walk approach, *Fibonacci Quart.* **57** (2019), 84–95.