ELEMENTARY PROBLEMS AND SOLUTIONS

EDITED BY
RUSS EULER AND JAWAD SADEK

Please submit all new problem proposals and their solutions to the Problems Editor, DR. RUSS EULER, Department of Mathematics and Statistics, Northwest Missouri State University, 800 University Drive, Maryville, MO 64468, or by email at reuler@nwmissouri.edu. All solutions to others' proposals must be submitted to the Solutions Editor, DR. JAWAD SADEK, Department of Mathematics and Statistics, Northwest Missouri State University, 800 University Drive, Maryville, MO 64468.

If you wish to have receipt of your submission acknowledged, please include a self-addressed, stamped envelope.

Each problem and solution should be typed on separate sheets. Solutions to problems in this issue must be received by November 15, 2010. If a problem is not original, the proposer should inform the Problem Editor of the history of the problem. A problem should not be submitted elsewhere while it is under consideration for publication in this Journal. Solvers are asked to include references rather than quoting "well-known results".

BASIC FORMULAS

The Fibonacci numbers F_n and the Lucas numbers L_n satisfy

$$F_{n+2} = F_{n+1} + F_n,\ F_0 = 0,\ F_1 = 1;$$

$$L_{n+2} = L_{n+1} + L_n,\ L_0 = 2,\ L_1 = 1.$$

Also, $\alpha = (1+\sqrt{5})/2,\ \beta = (1-\sqrt{5})/2,\ F_n = (\alpha^n - \beta^n)/\sqrt{5}$, and $L_n = \alpha^n + \beta^n$.

PROBLEMS PROPOSED IN THIS ISSUE

B-1066 Proposed by Hideyuki Ohtsuka, Saitama, Japan

Determine the value of

$$\sqrt{1+F_2\sqrt{1+F_4\sqrt{1+F_6\sqrt{\cdots\sqrt{1+F_{2n}\sqrt{\cdots}}}}}}.$$

B-1067 Proposed by N. Gauthier, Royal Military College of Canada, Kingston, ON, Canada

Let n be a positive integer. Find a closed-form expression for

$$\sum_{k=1}^{n} kF_k^3.$$

B-1068 Proposed by Mohammad K. Azarian, University of Evansville, Indiana.

If the sequence $s_0, s_1, s_2, s_3, \ldots$ is defined by the difference equation

$$2ks_{k+1} + (1-2k)s_k - s_{k-1} = 0, (k \geq 1), s_0 = F_n, s_1 = F_{2n},$$

then write $\lim_{k\to\infty} s_k$ in terms of Lucas numbers.

B-1069 Proposed by Pantelimon George Popescu, Politehnica University, Bucharest, Romania and José Luis Díaz-Barrero, Polytechnical University of Catalonia, Barcelona, Spain.

Let n, a, b, c, d be positive integers and A be a square matrix for which

$$A^n = \begin{pmatrix} F_{a+n-1} & L_{b+n-1} \\ L_{c+n-1} & F_{d+n-1} \end{pmatrix}.$$

Show that $b = c$ and $A = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$ or $A = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}$.

B-1070 Proposed by Roman Witula and Damian Slota, Silesian University of Technology, Poland.

Prove that

$$\left|\tan\left(x + \frac{1}{2}\arctan 2\right)\right| \cdot \left|\frac{\alpha - \tan x}{\beta - \tan x}\right| \equiv \alpha,$$

for all $x \neq (2k-1)\frac{\pi}{2}$, $x \neq k\pi + \arctan\beta$, $x \neq k\pi + \arctan\alpha$, $x \neq k\frac{\pi}{2} - \frac{1}{2}\arctan 2$ and $k \in \mathbb{Z}$.

SOLUTIONS

Because of deadline conflicts and to give individual solvers adequate time to solve recent problem proposals, we will not publish the Solutions section of this column in this issue. The solutions to problems B-1051 to B-1055 will appear in the August 2010 issue.