ADVANCED PROBLEMS AND SOLUTIONS

EDITED BY
FLORIAN LUCA

Please send all communications concerning ADVANCED PROBLEMS AND SOLUTIONS to FLORIAN LUCA, IMATE, UNAM, AP. POSTAL 61-3 (XANGARI), CP 58 089, MORELIA, MICHOACAN, MEXICO, or by e-mail at flucamatmor.unam.mx as files of the type tex, dvi, ps, doc, html, pdf, etc. This department especially welcomes problems believed to be new or extending old results. Proposers should submit solutions or other information that will assist the editor. To facilitate their consideration, all solutions sent by regular mail should be submitted on separate signed sheets within two months after publication of the problems.

PROBLEMS PROPOSED IN THIS ISSUE

H-685 Proposed by N. Gauthier, Kingston, ON

For k a positive integer prove the following identities:

a) $\sum_{m=1}^{k}\binom{2k-m-1}{k-1}(F_{2m}+F_m)=F_{3k}$;

b) $\sum_{m=1}^{k}\frac{m}{k}\binom{2k-m-1}{k-1}(F_{2m+2}-F_{m+1})=F_{3k}$;

c) $\sum_{m=1}^{k}\frac{2^m}{2^{2k}}\binom{2k-m-1}{k-1}(F_{2m}+(-1)^{m+1}F_m)=F_k$;

d) $\sum_{m=1}^{k}\frac{2^m m}{2^{2k+1}k}\binom{2k-m-1}{k-1}(F_{2m+2}+(-1)^{m+1}F_{m+1})=F_{3k}$.

H-686 Proposed by José Luis Díaz-Barero, Barcelona, Spain

Let n be a positive integer. Compute

$$\sum_{1\le i<j\le n} F_iF_j(F_i-F_j)^2.$$

H-687 Proposed by G. C. Greubel, Newport News, VA

i) Show that

$$\sum_{n=0}^{\infty}\left[\frac{1}{5n+1}-\frac{\beta^2}{5n+3}-\frac{\beta^4}{5n+4}-\frac{\beta^5}{5n+5}\right](-\beta^5)^n=\pi\left(\frac{\alpha^6}{5^5}\right)^{\frac{1}{4}}.$$

ii) From the series in i) and H-669 (corrected) show that

ii.1) $\sum_{n=0}^{\infty}\left[\frac{1}{5n+1}+\frac{1}{5n+2}-\frac{\beta^2}{5n+4}-\frac{\beta^4}{5n+5}\right]\left(-\beta^5\right)^n=\pi\left(\frac{\alpha^2}{5}\right)^{\frac{5}{4}}$;

ii.2) $$\sum_{n=0}^{\infty}\left[\frac{\alpha^3}{5n+2}+\frac{\alpha}{5n+3}-\frac{1}{5n+4}-\frac{1}{5n+5}\right]\left(-\beta^5\right)^n=\pi\left(\frac{\alpha^{14}}{5^5}\right)^{\frac{1}{4}};$$

ii.3) $$\sum_{n=0}^{\infty}\left[\frac{1}{5n+1}+\frac{\beta^2}{5n+2}+\frac{\beta^3}{5n+3}+\frac{\beta^3}{5n+4}\right](-\beta^5)^n=2\pi\left(\frac{\alpha^2}{5^5}\right)^{\frac{1}{4}}.$$

H-688 Proposed by Apoloniusz Tyszka, Krakow, Poland

Let

$$E_n=\{x_i=1,\ x_i+x_j=x_k,\ x_i\cdot x_j=x_k\ :\ i,j,k\in\{1,2,\dots,n\}\}.$$

Prove or disprove: If a system $S\subseteq E_n$ has only finitely many integer solutions $(x_1,\dots,x_n)$, then all such integer solutions satisfy $|x_i|\le 2^{2^{n-1}}$ for $i=1,\dots,n$.

SOLUTIONS

Some kth order Fibonacci Limits

H-668 Proposed by A. Cusumano, Great Neck, NY (Vol. 46, No. 1, February 2008)

For each $k\ge 2$, let $(F_n^{(k)})_{n\ge1}$ be the kth order linear recurrence given by

$$F_{n+k}^{(k)}=\sum_{i=0}^{k-1}F_{n+i}^{(k)},\quad \mathit{for\ all}\ n\ge 1,$$

with $F_n^{(k)}=1$ for $n=1,\dots,k$. Prove the following:
(a) $R_k=\lim_{n\to\infty}F_{n+1}^{(k)}/F_n^{(k)}$ exists for all $k\ge1$.
(b) $\lim_{k\to\infty}R_k=2$.
(c) $\lim_{k\to\infty}(R_{k+1}-R_k)/(R_{k+2}-R_{k+1})=2$.

Editor's comment. After this problem appeared in print, some solvers noted that this is the same as Problem H-197 proposed and solved by Lawrence Somer in the Fibonacci Quarterly of 1972 (solution in 1974). Problem H-197 is more general in the sense that the first k terms of the kth-order linear recurrence $(F_n^{(k)})_{n\ge1}$ were not necessarily assumed to be 1. The editor apologizes for this oversight.

Solved by Paul S. Bruckman and jointly by Ángel Plaza, Sergio Falcón and José M. Pacheco.

The Bilateral Binomial Theorem and Fibonacci Numbers

H-669 Proposed by G. C. Greubel, Newport News, VA (Vol. 46, No. 2, May 2008)

Show that

$$\sum_{n=0}^{\infty}\left[\frac{1}{5n+1}+\frac{2}{5n+2}+\frac{\beta^2}{5n+3}+\frac{\beta}{5n+4}-\frac{\beta^2}{5n+5}\right](-1)^n\beta^{5n}=\pi\left(\frac{\alpha^2}{5}\right)^{\frac{3}{4}}.$$

Solution by the proposer

We start with the series

$$S(\theta) = \sum_{n=1}^{\infty} \frac{\cos(4n-3)\theta}{n}(2\cos\theta)^n. \tag{1}$$

If we write it out term by term, we get

$$\begin{aligned} S(\theta) &= \frac{1}{1}\cos\theta\,(2\cos\theta)^1 + \frac{1}{2}\cos 5\theta\,(2\cos\theta)^2 + \frac{1}{3}\cos 9\theta\,(2\cos\theta)^3 \\ &\quad + \frac{1}{4}\cos 13\theta\,(2\cos\theta)^4 + \frac{1}{5}\cos 17\theta\,(2\cos\theta)^5 + \cdots . \end{aligned}$$

If we let $\theta = 2\pi/5$, we are then led to

$$S\left(\frac{2\pi}{5}\right) = \sum_{n=0}^{\infty}\left[\frac{\beta\cos\left(\frac{2\pi}{5}\right)}{5n+1} - \frac{\beta^2}{5n+2} - \frac{\beta^3\cos\left(\frac{3\pi}{5}\right)}{5n+3} + \frac{\beta^4\cos\left(\frac{\pi}{5}\right)}{5n+4} + \frac{\beta^5\cos\left(\frac{4\pi}{5}\right)}{5n+5}\right](-1)^n\beta^{5n}.$$

Inserting the values

$$\cos\left(\frac{\pi}{5}\right) = \frac{\alpha}{2}, \qquad \cos\left(\frac{2\pi}{5}\right) = -\frac{\beta}{2}, \qquad \cos\left(\frac{3\pi}{5}\right) = \frac{\beta}{2}, \qquad \cos\left(\frac{4\pi}{5}\right) = -\frac{\alpha}{2},$$

in the above series, we get

$$2\alpha^2 S\left(\frac{2\pi}{5}\right) = \sum_{n=0}^{\infty}\left[\frac{1}{5n+1} + \frac{2}{5n+2} + \frac{\beta^2}{5n+3} + \frac{\beta}{5n+4} - \frac{\beta^2}{5n+5}\right](-1)^n\beta^{5n}, \tag{2}$$

and we recognize in the right hand side the series that we want to evaluate.

Alternatively, the series (1) may be expanded into

$$S(\theta) = \cos(3\theta)\sum_{n=1}^{\infty}\frac{(2\cos\theta)^n}{n}\cos(4n\theta) + \sin(3\theta)\sum_{n=1}^{\infty}\frac{(2\cos\theta)^n}{n}\sin(4n\theta). \tag{3}$$

In order to evaluate this last series, we consider the series expansion of the natural logarithm in the following way. Since

$$-\ln(1-x) = \sum_{n=1}^{\infty}\frac{x^n}{n},$$

by letting $x = re^{i\theta}$ with $r \in (0,1)$, we get

$$\begin{aligned} \sum_{n=1}^{\infty}\frac{r^n}{n}e^{in\theta} &= -\ln(1-re^{i\theta}) = -\ln(1 - r\cos\theta - ir\sin\theta) \\ &= -\frac{1}{2}\ln(1-2r\cos\theta+r^2) + i\tan^{-1}\left(\frac{r\sin\theta}{1-r\cos\theta}\right). \end{aligned}$$

Equating the real and complex parts of both sides yields the series identities:

$$\sum_{n=1}^{\infty}\frac{r^n}{n}\cos n\theta = -\frac{1}{2}\ln(1-2r\cos\theta+r^2); \tag{4}$$

$$\sum_{n=1}^{\infty}\frac{r^n}{n}\sin n\theta = \tan^{-1}\left(\frac{r\sin\theta}{1-r\cos\theta}\right). \tag{5}$$

Now letting $\theta = 4\phi$ and $r = 2\cos\phi$ in equations (4), we obtain

$$\sum_{n=1}^{\infty} \frac{\cos 4n\phi}{n}(2\cos\phi)^n = -\frac{1}{2}\ln(1 - 4\cos\phi\cos 4\phi + 4\cos^2\phi); \tag{6}$$

$$\sum_{n=1}^{\infty} \frac{\sin 4n\phi}{n}(2\cos\phi)^n = \tan^{-1}\left(\frac{2\cos\phi\sin 4\phi}{1 - 2\cos\phi\cos 4\phi}\right). \tag{7}$$

When $\phi = 2\pi/5$, then the cosine series reduces to

$$\begin{aligned}\sum_{n=1}^{\infty} \frac{(-\beta)^n}{n}\cos\left(\frac{8n\pi}{5}\right) &= -\frac{1}{2}\ln\left(1 - 4\cos\left(\frac{2\pi}{5}\right)\cos\left(\frac{8\pi}{5}\right) + 4\cos^2\left(\frac{2\pi}{5}\right)\right) \\ &= -\frac{1}{2}\ln\left(1 - 4\left(\frac{-\beta}{2}\right)\left(-\frac{\beta}{2}\right) + 4\left(-\frac{\beta}{2}\right)^2\right) \\ &= -\frac{1}{2}\ln(1 - \beta^2 + \beta^2) = -\frac{1}{2}\ln(1) = 0,\end{aligned}$$

while the sine series reduces to

$$\sum_{n=1}^{\infty} \frac{(-\beta)^n}{n}\sin\left(\frac{8n\pi}{n}\right) = \tan^{-1}\left(\frac{-2\beta}{2-\beta^2}\sin\left(\frac{8\pi}{5}\right)\right) = \tan^{-1}\left(\frac{\beta}{\sqrt{\alpha}}\cdot 5^{\frac{1}{4}}\right),$$

where in the above we used the fact that $\sin(8\pi/5) = -\frac{1}{2}\sqrt{\sqrt{5}\alpha}$.

Thus, letting $\theta = 2\pi/5$ in (3), we obtain

$$\begin{aligned}S\left(\frac{2\pi}{5}\right) &= \cos\frac{6\pi}{5}\sum_{n=1}^{\infty}\frac{(-\beta)^n}{n}\cos\frac{8n\pi}{5} + \sin\frac{6\pi}{5}\sum_{n=1}^{\infty}\frac{(-\beta)^n}{n}\sin\frac{8\pi}{5} \\ &= \sin\frac{6\pi}{5}\cdot\tan^{-1}\left(\frac{\beta}{\sqrt{\alpha}}\cdot 5^{\frac{1}{4}}\right) = -\frac{1}{2}\sqrt{-\beta}5^{\frac{1}{4}}\tan^{-1}\left(\frac{\beta}{\sqrt{\alpha}}\cdot 5^{\frac{1}{4}}\right),\end{aligned} \tag{8}$$

where we used the fact that $\sin(6\pi/5) = -\frac{1}{2}\sqrt{-\beta}\cdot 5^{\frac{1}{4}}$. Since $\sin(\pi/5) = -\frac{1}{2}\cdot 5^{1/4}\cdot\sqrt{-\beta}$ and $\cos(\pi/5) = \frac{\alpha}{2}$, we get that $\tan(-\pi/5) = 5^{1/4}\frac{\beta}{\sqrt{\alpha}}$. With this calculation, in (8) we get

$$S\left(\frac{2\pi}{5}\right) = -\frac{1}{2}\sqrt{-\beta}5^{\frac{1}{4}}\cdot\left(-\frac{\pi}{5}\right) = \frac{\pi}{2\sqrt{\alpha}\cdot 5^{3/4}}.$$

Now the desired relation follows from the last relation above and (2).

Also solved by Paul S. Bruckman and Kenneth B. Davenport.

Sums of Fibonomial Coefficients

H-670 Proposed by P. Bruckman, Sointula, Canada (Vol. 46, No. 2, May 2008)

Let $\begin{bmatrix} n \\ k \end{bmatrix}$ denote the standard Fibonomial coefficient $F_nF_{n-1}\cdots F_{n-k+1}/(F_1F_2\cdots F_k)$.

(a) Define the following sums:

$$A_n = \sum_{k=0}^{n}(-1)^{k(k+1)/2}\begin{bmatrix} n \\ k \end{bmatrix}F_k, \qquad B_n = \sum_{k=0}^{n}(-1)^{k(k-1)/2}\begin{bmatrix} n \\ k \end{bmatrix}.$$

Prove that for $n \geq 1$, $A_n = -F_nB_{n-1}$.

(b) Define

$$C_n = \sum_{k=0}^{n} (-1)^{k(k+1)/2} \begin{bmatrix} n \\ k \end{bmatrix} L_k.$$

Prove that $C_n = -(L_n - 2)B_{n-1}$.

Solution by Harris Kwong, SUNY Fredonia, Fredonia, NY

(a) From

$$\begin{bmatrix} n \\ k \end{bmatrix} = \frac{F_n F_{n-1} \cdots F_{n-k+1}}{F_1 F_2 \cdots F_k} \cdot F_k = F_n \cdot \frac{F_{n-1} F_{n-2} \cdots F_{n-k+1}}{F_1 F_2 \cdots F_{k-1}} = F_n \begin{bmatrix} n-1 \\ k-1 \end{bmatrix},$$

we obtain

$$A_n = F_n \sum_{k=0}^{n} (-1)^{k(k+1)/2} \begin{bmatrix} n-1 \\ k-1 \end{bmatrix}.$$

Since $\left[\begin{smallmatrix} n-1 \\ -1 \end{smallmatrix}\right] = 0$, this sum in effect starts with $k = 1$. By setting $j = k - 1$, we find

$$\sum_{k=0}^{n} (-1)^{k(k+1)/2} \begin{bmatrix} n-1 \\ k-1 \end{bmatrix} = \sum_{j=0}^{n-1} (-1)^{(j+1)(j+2)/2} \begin{bmatrix} n-1 \\ j \end{bmatrix} = -\sum_{j=0}^{n-1} (-1)^{j(j-1)/2} \begin{bmatrix} n-1 \\ j \end{bmatrix} = -B_{n-1},$$

which proves that $A_n = -F_n B_{n-1}$.

(b) The identity $F_n L_k = F_k L_n + 2(-1)^k F_{n-k}$ yields

$$\begin{bmatrix} n \\ k \end{bmatrix} L_k = \frac{F_{n-1} F_{n-2} \cdots F_{n-k+1}}{F_1 F_2 \cdots F_k} \left[F_k L_n + 2(-1)^k F_{n-k}\right] = \begin{bmatrix} n-1 \\ k-1 \end{bmatrix} L_n + 2(-1)^k \begin{bmatrix} n-1 \\ k \end{bmatrix}.$$

Hence,

$$C_n = L_n \sum_{k=0}^{n} (-1)^{k(k+1)/2} \begin{bmatrix} n-1 \\ k-1 \end{bmatrix} + 2 \sum_{k=0}^{n} (-1)^{k(k+3)/2} \begin{bmatrix} n-1 \\ k \end{bmatrix}.$$

The first sum is the same sum we obtained in the proof of part (a), and we can terminate the second sum at $k = n - 1$ because $\left[\begin{smallmatrix} n-1 \\ n \end{smallmatrix}\right] = 0$. Therefore,

$$\begin{aligned} C_n &= -L_n B_{n-1} + 2 \sum_{k=0}^{n-1} (-1)^{k(k+3)/2} \begin{bmatrix} n-1 \\ k \end{bmatrix} = -L_n B_{n-1} + 2 \sum_{k=0}^{n-1} (-1)^{k(k-1)/2} \begin{bmatrix} n-1 \\ k \end{bmatrix} \\ &= -L_n B_{n-1} + 2B_{n-1}, \end{aligned}$$

which completes the proof of (b).

Also solved by H. J. Seiffert and the proposer.

Sums of Fibonacci and Lucas Polynomials

H-671 Proposed by G. C. Greubel, Newport News, VA (Vol. 46, No. 2, May 2008)

Let $\phi_n(x, y)$ be the bi-variate Fibonacci and Lucas polynomials. Find expansions for

$$\sum_{n=0}^{\infty} \binom{n+m}{n} \phi_{n+p}(x, y),$$

in the form

$$P(x,y)+\sum_{r=0}^{\lambda}\binom{\lambda}{r}Q_r(x,y),$$

where $P(x,y)$ and $Q_r(x,y)$ are general polynomials and λ is given by: (A) $m+1$, (B) p.

Solution by the proposer

Let

$$S_1=S_1(x,y)=\sum_{n=0}^{\infty}\binom{n+m}{n}F_{n+p}(x,y). \tag{9}$$

By using the fact that

$$\sum_{n=0}^{\infty}\binom{n+m}{n}t^n=(1-t)^{-m-1},$$

we have

$$S_1=\left(\frac{1}{\alpha-\beta}\right)\left[\alpha^p(1-\alpha)^{-m-1}-\beta^p(1-\beta)^{-m-1}\right], \tag{10}$$

where $\alpha=\alpha(x,y)=\frac{1}{2}(x+\sqrt{x^2+4y})$ and $\beta=\beta(x,y)=\frac{1}{2}(x-\sqrt{x^2+4y})$. The term $1-\alpha(x,y)$ can be evaluated in the following way:

$$\begin{aligned}1-\alpha(x,y) &= 1-\frac{1}{2}(x+\sqrt{x^2+4y})=-\frac{1}{2}(x-2+\sqrt{x^2+4y})\\ &= -\frac{1}{2}(x-2+\sqrt{(x-2)^2+4(y+x-1)})=-\alpha(x-2,y+x-1). \quad (11)\end{aligned}$$

A similar relation holds for $1-\beta(x,y)$ and is given by

$$1-\beta(x,y)=-\beta(x-2,y+x-1). \tag{12}$$

With this, we have from equations (10) – (12)

$$S_1=\left(\frac{(-1)^{m+1}}{\alpha-\beta}\right)\left[\alpha^p(x,y)\alpha^{-m-1}(a,b)-\beta^p(x,y)\beta^{-m-1}(a,b)\right], \tag{13}$$

where $a=x-2$ and $b=y+x-1$. By using the relation $\alpha(x,y)\beta(x,y)=-y$, equation (13) becomes

$$S_1=\left(\frac{(y+x-1)^{-m-1}}{\sqrt{x^2+4y}}\right)\left[\alpha^p(x,y)\beta^{m+1}(a,b)-\beta^p(x,y)\alpha^{m+1}(a,b)\right]. \tag{14}$$

This last expression can be transformed as

$$\begin{aligned}\Phi &= \alpha^p(x,y)\beta^{m+1}(a,b)-\beta^p(x,y)\alpha^{m+1}(a,b)\\ &= (\alpha^p(x,y)-\beta^p(x,y))\left(\alpha^{m+1}(a,b)+\beta^{m+1}(a,b)\right)\\ &- \left[\alpha^p(x,y)\alpha^{m+1}(a,b)-\beta^p(x,y)\beta^{m+1}(a,b)\right].\end{aligned}$$

Let the bracketed terms be given by σ. With this is mind our expression (14) is seen to be

$$S_1=(y+x-1)^{-m-1}F_p(x,y)L_{m+1}(a,b)-\left(\frac{(y+x-1)^{-m-1}}{\sqrt{x^2+4y}}\right)\sigma. \tag{15}$$

The terms in σ can be expanded, with dependence upon m, in the following way. Consider the term $\alpha^p(x,y)\alpha^{m+1}(a,b)$. This takes the form

$$\alpha^p(x,y)\alpha^{m+1}(a,b) = \alpha^p(x,y)(-1)^{m+1}\left(1-\alpha(x,y)\right)^{m+1} = \sum_{r=0}^{m+1}\binom{m+1}{r}(-1)^{r+m+1}\alpha^{p+r}(x,y).$$

A similar expression holds for the term involving $\beta^p(x,y)\beta^{m+1}(a,b)$. Combining these results we have

$$\sigma = \sqrt{x^2+4y}\sum_{r=0}^{m+1}\binom{m+1}{r}(-1)^{r+m+1}F_{p+r}(x,y). \tag{16}$$

Using this expansion in equation (15), with the use of equations (16) and (9), provides the desired result

$$\begin{aligned}&\sum_{n=0}^{\infty}\binom{n+m}{n}F_{n+p}(x,y)\\ &= \mu(x,y)\left\{F_p(x,y)L_{m+1}(x-2,y+x-1)+\sum_{r=0}^{m+1}\binom{m+1}{r}(-1)^{r+m}F_{p+r}(x,y)\right\},\end{aligned}$$

where $\mu(x,y) = (y+x-1)^{-m-1}$. This expansion depends on the value of m. The expansion that depends upon p can be derived by reconsidering the terms in σ. Consider $\alpha^p(x,y)\alpha^{m+1}(a,b)$ in the following way:

$$\alpha^p(x,y)\alpha^{m+1}(a,b) = \alpha^{m+1}(a,b)\left(1+\alpha(a,b)\right)^p = \sum_{r=0}^{p}\binom{p}{r}\alpha^{m+r+1}(a,b).$$

Again a similar expansion for the term $\beta^p(x,y)\beta^{m+1}(a,b)$ can be obtained. With the new expansions σ becomes

$$\begin{aligned}\sigma &= \alpha^p(x,y)\alpha^{m+1}(a,b) - \beta^p(x,y)\beta^{m+1}(a,b) = \sum_{r=0}^{p}\binom{p}{r}\left[\alpha^{m+r+1}(a,b)-\beta^{m+r+1}(a,b)\right]\\ &= \sqrt{x^2+4y}\cdot\sum_{r=0}^{p}\binom{p}{r}F_{m+r+1}(a,b).\end{aligned} \tag{17}$$

By using equations (9), (15), and (17), we have the result

$$\begin{aligned}&\sum_{n=0}^{\infty}\binom{n+m}{n}F_{n+p}(x,y)\\ &= \mu(x,y)\left\{F_p(x,y)L_{m+1}(x-2,y+x-1)-\sum_{r=0}^{p}\binom{p}{r}F_{m+r+1}(x-2,y+x-1)\right\}.\end{aligned} \tag{18}$$

By following the same pattern as given above the results for the bi-variate Lucas polynomials are given by

$$\sum_{n=0}^{\infty}\binom{n+m}{n}L_{n+p}(x,y)$$
$$=\mu(x,y)\left[L_p(x,y)L_{m+1}(x-2,y+x-1)+\sum_{r=0}^{m+1}\binom{m+1}{r}(-1)^{r+m}L_{p+r}(x,y)\right],$$

and

$$\sum_{n=0}^{\infty}\binom{n+m}{n}L_{n+p}(x,y)$$
$$=\mu(x,y)\left[L_p(x,y)L_{m+1}(x-2,y+x-1)-\sum_{r=0}^{p}\binom{p}{r}L_{m+r+1}(x-2,y+x-1)\right].$$

In general, one can state:

$$\sum_{n=0}^{\infty}\binom{n+m}{n}\phi_{n+p}(x,y)$$
$$=\mu(x,y)\left[\phi_p(x,y)L_{m+1}(x-2,y+x-1)+\sum_{r=0}^{m+1}\binom{m+1}{r}(-1)^{r+m}\phi_{p+r}(x,y)\right],$$

and

$$\sum_{n=0}^{\infty}\binom{n+m}{n}\phi_{n+p}(x,y)$$
$$=\mu(x,y)\left[\phi_p(x,y)L_{m+1}(x-2,y+x-1)-\sum_{r=0}^{p}\binom{p}{r}\phi_{m+r+1}(x-2,y+x-1)\right].$$

Also solved by Paul S. Bruckman.

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