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Send all communications concerning Advanced Problems and Solutions to Verner E. Hoggatt, Jr., Mathematics Department, San Jose State College, San Jose, Calif. This department especially welcomes problems believed to be new or extending old results. Proposers should submit solutions or other information that will assist the editor. To facilitate their consideration, solutions should be submitted on separate signed sheets within two months after publication of the problems.

H-1. Proposed by H. W. Gould, West Virginia University, Morgantown, W. Va.

Find a formula for the nth non-Fibonacci number, that is, for the sequence 4, 6, 7, 9, 10, 11, 12, 14, 15, 16, 17, 18, 19, 20, 22, 23, ...
(See paper by L. Moser and J. Lambek, American Mathematical Monthly, vol. 61 (1954), pp. 454-458).

H-2. Proposed by L. Moser and L. Carlitz, University of Alberta, Edmonton, Alberta, and Duke University, Durham, N.C. (See also C.S. Ogilvy: Tomorrow's Mathematics, p. 100).

Resolve the conjecture: There are no Fibonacci numbers which are integral squares except 0, 1, and 144.

H-3. Proposed by Verner E. Hoggatt, Jr., San Jose State College, San Jose, Calif.

Show $F_{2n-2} < F_n^2 < F_{2n-1}$, $\quad n \geq 3$;

$$F_{2n-1} < L_{n-1}^2 < F_{2n}, \quad n \geq 4,$$

where F_n and L_n are the nth Fibonacci and nth Lucas numbers, respectively.

H-4. Proposed by I. D. Ruggles, San Jose State College, San Jose, Calif.

Prove the identity:

$$F_{r+1}F_{s+1}F_{t+1} + F_rF_sF_t - F_{r-1}F_{s-1}F_{t-1} = F_{r+s+t}$$

Are there any restrictions on the integral subscripts?

H-5. Proposed by Terry Brennan, Lockheed Missiles Space Co., Sunnyvale, Calif.

(i) If $$[{}_mF_n] = \frac{(F_mF_{m-1}\cdots F_1)}{(F_nF_{n-1}\cdots F_1)(F_{m-n}F_{m-n-1}\cdots F_1)}$$

then $2\,[{}_mF_n] = [{}_{m-1}F_n]\,L_n + [{}_{m-1}F_{n-1}]\,L_{m-n}$,

where F_n and L_n are the nth Fibonacci and the nth Lucas numbers, respectively.

(ii) Show that this generalized binomial coefficient $[{}_mF_n]$ is always an integer.

H-6. Proposed by Brother U. Alfred, St. Mary's College, Calif.

Determine the last three digits, in base seven, of the millionth Fibonacci number.

H-7. Proposed by Verner E. Hoggatt, Jr., San Jose State College, San Jose, Calif.

If F_n is the nth Fibonacci number find

$$\lim_{n \to \infty} \sqrt[n]{F_n} = L$$

and show that

$$\sqrt[2n]{\sqrt{5}\,F_{2n}} < L < \sqrt[2n+1]{\sqrt{5}\,F_{2n+1}} \qquad \text{for } n \geq 2.$$

H-8. Proposed by Brother U. Alfred, St. Mary's College, Calif.

Prove

$$\begin{vmatrix} F_n^2 & F_{n+1}^2 & F_{n+2}^2 \\ F_{n+1}^2 & F_{n+2}^2 & F_{n+3}^2 \\ F_{n+2}^2 & F_{n+3}^2 & F_{n+4}^2 \end{vmatrix} = 2(-1)^{n+1},$$

where F_n is the nth Fibonacci number.

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FIBONACCI ARTICLES SOON TO APPEAR

A.F. Horadam, Complex Fibonacci Numbers and Fibonacci Quaternions, The American Mathematical Monthly.

S.L. Basin, The Appearance of Fibonacci Numbers and the Q-Matrix in Electrical Network Theory, Mathematics Magazine, March, 1963.

S.L. Basin, An Application of Continuants as a Link between Chebyshev and Fibonacci, Mathematics Magazine

S.L. Basin, Generalized Fibonacci Numbers and Squared Rectangles, American Mathematical Monthly.

D. Zeitlin, On Identities for Fibonacci Numbers. Classroom Notes, American Mathematical Monthly.

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