THE FIBONACCI SEQUENCE AS IT APPEARS IN NATURE

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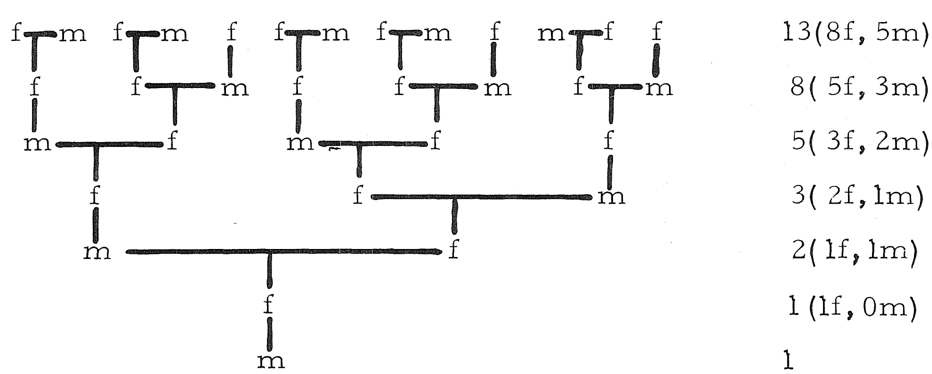
1. INTRODUCTION

The regular spiral arrangement of leaves around plant stalks has enjoyed much attention by botanists and mathematicians in their attempt to unravel the mysteries of this organic symmetry. Because of the abundance of literature on phyllotaxis no more attention will be devoted to it here. However, the Fibonacci numbers have the strange habit of appearing where least expected in other natural phenomena. The following snapshots will demonstrate this fact. (See references 1 and 3 for a discussion of phyllotaxis.)

2. THE GENEALOGICAL TREE OF THE MALE BEE

We shall trace the ancestral tree of the male bee backwards, keeping in mind that the male bee hatches from an unfertilized egg. The fertilized eggs hatch into females, either workers or queens.

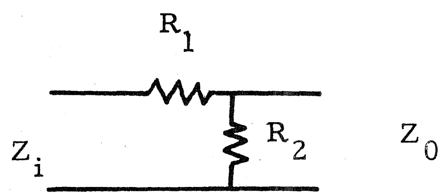
The following diagram clearly shows that the number of ancestors in any one generation is a Fibonacci number. The symbol (m) represents a male and the symbol (f) represents a female.



3. SIMPLE ELECTRICAL NETWORKS

Even those people interested in electrical networks cannot escape from our friend Fibonacci. Consider the following simple network of resistors known as a ladder network. This circuit consists of n L-sections in cascade and can be characterized or described by calculating the <u>attenuation</u> which is simply the input voltage divided by the output voltage and denoted by A, the <u>input impedance</u> Z_i and the <u>output impedance</u> Z_0. (For a detailed discussion refer to reference 4.)

Proceeding in a manner similar to mathematical induction, consider the following ladder networks.

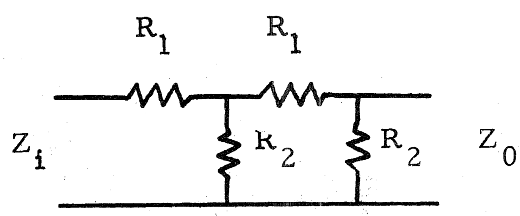


When $n = 1$, $Z_0 = R_2$

$$Z_i = R_1 + R_2$$

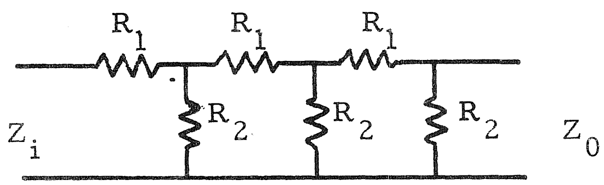
$$A = R_1/R_2 + 1.$$

When n = 2:



$$Z_0 = \frac{R_2(R_1 + R_2)}{R_1 + 2R_2}, \quad A = \frac{(R_1 + R_2)(R_1 + 2R_2) - R_2^2}{R_2^2}$$

$$Z_i = \frac{R_1R_2(R_1 + 2R_2) + R_2(R_1 + R_2)}{R_1 + 2R_2}$$



When n = 3:

$$Z_0 = \frac{R_1R_2(R_1 + 2R_2) + R_2^2(R_1 + R_2)}{(R_1 + R_2)(R_1 + 3R_2)}$$

$$Z_i = \frac{R_1^3 + 5R_1^2R_2 + 6R_1R_2^2 + R_2^3}{R_1^2 + 4R_1R_2 + 3R_2^2}$$

$$A = \frac{R_1^3 + 5R_1^2R_2 + 6R_1R_2^2 + R_2^3}{R_2^2}$$

Now suppose all the resistors have the same value, namely, $R_1 = R_2 = 1$ ohm. We have by induction:

$$Z_0 = \frac{F_{2n-1}}{F_{2n}}, \qquad A = (F_{2n-1} + F_{2n}) = F_{2n+1},$$

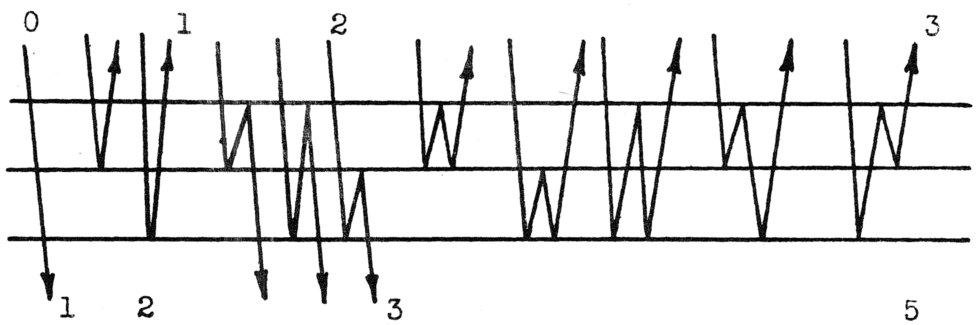
$$Z_i = \frac{F_{2n+1}}{F_{2n}}$$

In other words, the ladder network can be analyzed by inspection; as n is allowed to increase, n = 1, 2, 3, 4, . . . , the value of Z_0 for n L-sections coincides with the nth term in the sequence of Fibonacci ratios, i.e., 1/1, 2/3, 5/8, 13/21, ... The value for A is given by the sum of the numerator and denominator of Z_0. The value of Z_i is also clearly related to the expression for A and Z_0.

4. SOME REFLECTIONS (Communicated to us by Leo Moser)

The reflection of light rays within two plates of glass is expressed in terms of the Fibonacci numbers, i.e., if no reflections are allowed, one ray will emerge; if one reflection is allowed, two rays will emerge, ..., etc.

Number of reflections



Number of emerging rays

Refer to problem B-6 of Elementary Problems and Solutions.

For Additional Reading

1. Introduction to Geometry, H.S.M. Coxeter, pp. 169-172. John Wiley and Sons, Inc., 1961. A complete chapter on phyllotaxis and Fibonacci numbers appears in easily digestible treatment.

2. The Fibonacci Numbers, N.N. Vorobyov, Blaisdell, New York, 1961. (Translation from the Russian by Halina Moss)

This booklet discusses the elementary properties of Fibonacci numbers, their application to geometry, and their connection with the theory of continued fractions.

3. The Language of Mathematics, Robert Land, Chapter XIII, pp. 215-225, John Murray, 1960, London. A very interesting chapter including some phyllotaxis.

4. Appearance of Fibonacci Numbers and the Q Matrix in Electrical Network Theory, S.L. Basin, Mathematics Magazine, March, 1963.