Edited by S.L. Basin,
San Jose State College

Send all communications regarding Elementary Problems and Solutions to S.L. Basin, 946 Rose Ave., Redwood City, California. We welcome any problems believed to be new in the area of recurrent sequences as well as new approaches to existing problems. The proposer must submit his problem with solution in legible form, preferably typed in double spacing, with the name(s) and address of the proposer clearly indicated. Solutions should be submitted within two months of the appearance of the problems.

B-1. Proposed by I.D. Ruggles, San Jose State College, San Jose, Calif.

Show that the sum of twenty consecutive Fibonacci numbers is divisible by F_{10}.

B-2. Proposed by Verner E. Hoggatt, Jr., San Jose State College, San Jose, Calif.

Show that

$$u_{n+1} + u_{n+2} + \cdots + u_{n+10} = 11\, u_{n+7}$$

holds for generalized Fibonacci numbers such that $u_{n+2} = u_{n+1} + u_n$, where $u_1 = p$ and $u_2 = q$.

B-3. Proposed by J.E. Householder, Humboldt State College, Arcata, Calif.

Show that $F_{n+24} \equiv F_n \pmod 9$,

where F_n is the nth Fibonacci number.

B-4. Proposed by S. L. Basin and Vladimir Ivanoff, San Jose State College and San Carlos, Calif.

Show that $\sum_{i=0}^{n} \binom{n}{i} F_i = F_{2n}$.

Generalize.

B-5. Proposed by L. Moser, University of Alberta, Edmonton, Alberta.

Show that, with order taken into account, in getting paid an integral number n dollars, using only one-dollar and two-dollar bills, that the number of different ways is F_{n+1} where F_n is the nth Fibonacci number.

B-6. Proposed by L. Moser and M. Wyman, University of Alberta.

Light rays fall upon a stack of two parallel plates of glass, one ray goes through without reflection, two rays (one from each internal interface opposing the ray) will be reflected once but in different ways, three will be reflected twice but in different ways. Show that the number of distinct paths, which are reflected exactly n times, is F_{n+2}.

B-7. Proposed by H. W. Gould, West Virginia University, Morgantown, West Va.

Show that $\dfrac{x(1-x)}{1-2x-2x^2+x^3} = \sum_{i=0}^{\infty} F_i^2 x^i$.

Is the expansion valid at $x = 1/4$? That is, does

$$\sum_{i=0}^{\infty} F_i^2 / 4^i = 12/25\ ?$$

B-8. Proposed by J. A. Maxwell, Stanford University.

Show that

$$\text{(i)}\quad F_{n+1}2^n + F_n 2^{n+1} \equiv 1 \pmod 5$$

$$\text{(ii)}\quad F_{n+1}3^n + F_n 3^{n+1} \equiv 1 \pmod{11}$$

$$\text{(iii)}\quad F_{n+1}5^n + F_n 5^{n+1} \equiv 1 \pmod{29}.$$

Generalize.

****

RESEARCH CONFERENCE of the FIBONACCI ASSOCIATION

On December 15, 1962, the Fibonacci Association held its first research conference at San Jose State College. Papers delivered on the occasion were as follows:

Some Determinants Involving Powers of Fibonacci Numbers . . . . Brother U. Alfred

Some Proofs of Conjectures in Brother Alfred's Paper . . . . Terry Brennan

Squaring Rectangles Using Generalized Fibonacci Numbers . . . . Stanley L. Basin

The Period of the Ratio of Fibonacci Sequence Modulo M . . . . .John E. Vinson

Representations by Complete Fibonacci Sequences . . . . Verner E. Hoggatt, Jr.

Fibonacci Matrices . . . . . . James A. Maxwell

A Ray Incident on N Flats . . . . . Bjarne Junge