ADVANCED PROBLEMS AND SOLUTIONS

EDITED BY VERNER E. HOGGATT, JR., SAN JOSE STATE COLLEGE

Send all communications concerning Advanced Problems and Solutions to Verner E. Hoggatt, Jr., Mathematics Department, San Jose State College, San Jose, Calif. This department especially welcomes problems believed to be new or extending old results. Proposers should submit solutions or other information that will assist the editor. To facilitate their consideration, solutions should be submitted on separate signed sheets within two months after publication of the problems.

H-9 *Proposed by Olga Taussky, California Institute of Technology, Pasadena, California*

Find the numbers $a_{n,r}$, where $n \geq 0$ and r are integers, for which the relations

$$a_{n,r} + a_{n,r-1} + a_{n,r-2} = a_{n+1,r}$$

and

$$a_{0,r} = \delta_{o,r} = \begin{cases} 0 & r \neq 0 \\ 1 & r = 0 \end{cases}$$

hold.

H-10 *Proposed by R. L. Graham, Bell Telephone Laboratories, Murray Hill, New Jersey*

Show that $$\sum_{n=1}^{\infty} \frac{1}{F_n} = 3 + \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{F_n F_{n+1} F_{n+2}}$$

H-11 *Proposed by John L. Brown, Jr., Ordnance Research Laboratory, The Pennsylvania State University, University Park, Penna.*

Find the function whose formal Fourier series is

$$f(x) = \sum_{n=1}^{\infty} \frac{F_n \sin nx}{n!},$$

where F_n is the nth Fibonacci number.

H-12 *Proposed by D. E. Thoro, San Jose State College, San Jose, California*

Find a formula for the nth term in the sequence:

$$1, 3, 4, 6, 8, 9, 11, 12, 14, 16, 17, 19, 21, 22, 24, 25, \quad .$$

H-13 *Proposed by H. W. Gould, West Virginia University, Morgantown, W. Va., and Verner E. Hoggatt, Jr., San Jose State College, San Jose, Calif.*

Show that

$$F_n = \sum_{j=0}^{r} \binom{r}{j} F_{k-1}^{r-j} F_k^j F_{n+j-rk} \quad .$$

H-14 *Proposed by David Zeitlin, Minneapolis, Minnesota, and F. D. Parker, University of Alaska, College, Alaska.*

Prove the Fibonacci identity

$$F_{n+4}^3 - 3F_{n+3}^3 - 6F_{n+2}^3 + 3F_{n+1}^3 + F_n^3 = 0$$

H-15 *Proposed by Malcolm H. Tallman, Brooklyn, New York*

Do there exist integers N_1, N_2, and N_3 for which the following expressions cannot equal other Fibonacci numbers?

$$\text{(i)} \qquad F_n^3 - F_n^2 F_m - F_m^3 \qquad m, n \geq N_1 \;,$$

$$\text{(ii)} \qquad F_n^3 + F_n^2 F_m + F_n F_m^2 \qquad m, n \geq N_2 \;,$$

$$\text{(iii)} \qquad F_n^2 - 3F_m^3 \qquad m, n \geq N_3 \quad .$$

H-16 *Proposed by H. W. Gould, West Virginia University, Morgantown, West Virginia*

Define the ordinary Hermite polynomials by $H_n = (-1)^n e^{x^2} D^n (e^{-x^2})$.

Show that:

$$\text{(i)} \qquad \sum_{n=0}^{\infty} H_n(x/2) \frac{x^n}{n!} = 1 \quad ,$$

$$\text{(ii)} \qquad \sum_{n=0}^{\infty} H_n(x/2) \frac{x^n}{n!} F_n = 0 \quad ,$$

$$\text{(iii)} \qquad \sum_{n=0}^{\infty} H_n(x/2) \frac{x^n}{n!} L_n = 2e^{-x^2} \quad ,$$

where F_n and L_n are the nth Fibonacci and nth Lucas numbers, respectively.

H-17 *Proposed by Brother U. Alfred, St. Mary's College, Calif.*

Sum: $$\sum_{k=1}^{n} k^3 F_k$$

H-18 *Proposed by R. G. Buschman, Oregon State University, Corvallis, Ore.*

"Symbolic relations" are sometimes used to express identities. For example, if F_n and L_n denote, respectively, Fibonacci and Lucas numbers, then

$$(1 + L)^n \doteq L_{2n}, \qquad (1 + F)^n \doteq F_{2n}$$

are known identities, where $\doteq$ denotes that the exponents on the symbols are to be lowered to subscripts after the expansion is made.

(a) Prove $(L + F)^n = (2F)^n$.

(b) Evaluate $(L + L)^n$.

(c) Evaluate $(F + F)^n$.

(d) How can this be suitably generalized?

NOTE: On occasion there will be problems listed at the ends of the articles in the advanced and elementary sections of the magazine. These problems are to be considered as logical extensions of the corresponding problem sections and solutions for these problems will be discussed in these sections as they are received.

See, for example, "Expansion of Analytic Functions In Polynomials Associated with Fibonacci Numbers," by Paul F. Byrd, San Jose State College, in the first issue of the Quarterly, and "Linear Recurrence Relations – Part I," by James Jeske, San Jose State College, in this issue.

Solutions for problems in ISSUE ONE will appear in ISSUE THREE.