1.5 Prove that there are infinitely many primes. Hint: Assuming that p_n is the largest prime, Euclid considered the expression $N = 1 + 2 \cdot 3 \cdot 5 \cdot 7 \cdots p_n$. Now either N is prime or N is composite. Complete his proof by investigating the consequences of each alternative.

Additional hints may be found on p. 80.

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FIBONACCI FORMULAS

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If you have a favorite Fibonacci formula, send it to us and we will try to publish it. Some historically interesting ones are shown below.

1. Perhaps the first Fibonacci formula was developed by Simpson in 1753.

$$F_{n+1} F_{n-1} - F_n^2 = (-1)^n$$

2. A very important formula was developed in 1879 by an obscure French mathematician, Aurifeuille. In fact, it is his one claim to fame.

$$L_{5n} = L_n (L_{2n} - 5F_n + 3)(L_{2n} + 5F_n + 3)$$

3. The only formula involving cubes of Fibonacci numbers given in Dickson's "History of the Theory of Numbers" is due to Lucas.

$$F_{n+1}^3 + F_n^3 - F_{n-1}^3 = F_{3n}$$

The late Jekuthiel Ginsburg offers $F_{n+2}^3 - 3F_n^3 + F_{n-1}^3 = 3F_{3n}$.

4. The recursion formula for sub-factorials is similar to the one for Fibonacci numbers: $P_{n+1} = n(P_n + P_{n-1})$; $P_0 = 1$, $P_1 = 0$.

5. Fibonacci numbers have been related to almost every other kind of number. Here is H. S. Vandiver's relation with Bernoulli numbers.

$$\sum_{k=0}^{\overline{p-3}} B_{2k} F_{2k} \equiv \frac{1}{2} \pmod p \quad \text{if } p = 5a \pm 1$$

$$\sum_{k=0}^{\overline{p-3}} B_{2k} F_{2(k-1)} \equiv 1 \pmod p \quad \text{if } p = 5a \pm 2$$

$\overline{p-3}$ denotes the greatest integer not exceeding $(p - 3)/2$.

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I think that this is a good idea.

Ed.