A Computer Investigation of a Property of the Fibonacci Sequence

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Publication of a table of the first 571 Fibonacci numbers in Recreational Mathematics Magazine (Oct. 1962) brought out the fact that the last (units) digit of the sequence is periodic with period 60, i.e., the 1, 1, 2, 3, 4, ··· sequence repeats on the last digit every 60 entires of the sequence. It also appeared that the last two are similarly periodic with a period of 300. Noting that the table had been calculated by an IBM 7090 digital computer, I resolved to set up our IBM 1620 to check out the above observations and extend to more digits. The size of our memory (20K) prohibited calculation of the terms of the sequence in their entirety, but this was not necessary since it was quite easy on this machine to truncate off all the digits of the running sums beyond those under consideration. The machine verified that the last two digits repeat every 300 times, the last three every 1500, the last four every 15000, the last five every 150,000, and finally after the computer ran for nearly three hours a repetition of the last six digits appeared at the 1,500,000th Fibonacci number. These may be written in the form:

$$F_{(n+60)} - F_n \equiv 0 \pmod{10}$$
$$F_{(n+300)} - F_n \equiv 0 \pmod{100}$$
$$F_{(n+1500)} - F_n \equiv 0 \pmod{1000}$$
$$F_{(n+15000)} - F_n \equiv 0 \pmod{10000}$$
$$F_{(n+150000)} - F_n \equiv 0 \pmod{100000}$$
$$F_{(n+1500000)} - F_n \equiv 0 \pmod{1000000}$$

There does not yet seem to be any way of guessing the next period, but perhaps a new program for the machine which will permit initialization at any point in the sequence for a test will cut down computer time enough so that more data can be gathered for conjecturing some rule for these repetition periods.