$$\beta^p \beta^{n-p-1} + \beta^{p-1} \beta^{n-p} = \beta^{n+1} + \beta^{n-1} = \beta^n(\beta - \alpha)$$

and

$$\beta^p \alpha^{n-p+1} + \beta^{p-1} \alpha^{n-p} = 0$$

we obtain

$$\beta^n = \beta^p F_{n-p+1} + \beta^{p-1} F_{n-p} .$$

Similarly, one can show that

$$\alpha^n = \alpha^p F_{n-p+1} + \alpha^{p-1} F_{n-p} .$$

It then follows that $F_n = F_p F_{n-p+1} + F_{p-1} F_{n-p}$ and if $\{u_n\}$ is an F-sequence, then

$$u_n = u_p F_{n-p+1} + u_{p-1} F_{n-p} .$$

Note that if $q = n - p + 1$, then $u_{p+q-1} = u_p F_q + u_{p-1} F_{q-1}$.

Since $\beta^n - \alpha^n = \sqrt{5} F_n$ and $\beta^n + \alpha^n = L_n$, it follows that

$$\beta^n = \frac{L_n + \sqrt{5} F_n}{2}$$

and

$$\alpha^n = \frac{L_n - \sqrt{5} F_n}{2} .$$

HINTS TO BEGINNERS' CORNER PROBLEMS

(See page 59)

1.1 Examine $\frac{n}{p}$.

1.2 Use identity III.

1.3 Notice that p, $p + 1$, $p + 2$ are three consecutive integers. Since $p > 3$ is an odd prime, $p + 1$ is even. Why must $p + 1$ be a multiple of 3?

1.4 $2^{5 \cdot 7} - 1 = (2^5)^7 - (1)^7 = (2^5 - 1)\,[(2^5)^6 + (2^5)^5 + \cdots + (2^5) + 1]$.

1.5 If N is composite, then by T1 it must have a prime factor p. This factor must be one of the following: $2, 3, 5, 7, \cdots, p_n$. Thus $p|N$ and $p|(2 \cdot 3 \cdot 5 \cdots p_n)$.