LEONARDO FIBONACCI

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The Fibonacci Quarterly receives its name from Leonardo of Pisa (or Leonardo Pisano), better known as Leonardo Fibonacci (Fibonacci is a contraction of Filius Bonacci, son of Bonacci). Leonardo was born about 1175 in the commercial center of Pisa. This was a time of great interest and importance in the history of Western Civilization. One finds the influence of the crusades stirring and awakening the people of Europe by bringing them in contact with the more advanced intellect of the East. During this time the Universities of Naples, Padua, Paris, Oxford, and Cambridge were established, the Magna Carta signed in England, and the long struggle between the Papacy and the Empire was culminated. Commerce was flourishing in the Mediterranean world and adventurous travelers such as Marco Polo were penetrating far beyond the borders of the known world.

It is in this growing commercial activity that we find the young Leonardo at Bugia on the Northern coast of Africa. Here the merchants of Pisa and other commercial cities of Italy had large warehouses for the storage of their goods. Actually very little is known about the life of this great mathematician. No contemporary historian makes mention of him, and one must look to his writings to find information about him. In the preface of his first and most important work, Liber Abbaci (I), Leonardo tells us that his father, the head of one of the warehouses of Bugia, instructed him to study arithmetic. In Bugia, he received his early education from a Moorish schoolmaster.

Leonardo then traveled about the Mediterranean visiting Egypt, Syria, Greece, Sicily, southern France, and Constantinople. He met with scholars and studied the various systems of arithmetic then in use. Leonardo was persuaded that the Hindu-Arabic system was superior to the methods then adopted in the different countries he had visited and that it was even superior to the Algorithma and the method of Pythagorus. He busied himself with the subject and carried on his own research, intent upon bringing the Hindu-Arabic system to his Italian countrymen. The study and research in mathematics so absorbed him that he seems to have devoted his life to this pursuit and spent little time in commerce which was flourishing at that time and was the favorite occupation of his fellow citizens. Yet most of the applications Leonardo makes in his works are in the field of commerce. In one place, he gives a careful evaluation of the money systems of the countries of his travels.

Leonardo returned to Italy about 1200 and in 1202 wrote Liber Abbaci (I), in which he gave a thorough treatment of arithmetic and algebra, the first that had been written by a Christian. The work is divided into 15 chapters. The chapter contents are given here to indicate the scope of the work: (1) Reading and writing numbers in the Hindu-Arabic system; (2) Multiplication of integers; (3) Addition of integers; (4) Subtraction of integers; (5) Division of integers; (6) Multiplication of integers by fractions; (7) Additional work with fractions; (8) Prices of goods; (9) Barter; (10) Partnership; (11) Alligation; (12) Solutions of problems; (13) Rule of false position; (14) Square and cube roots; (15) Proportions, and Geometry and algebra.

The last and most important chapter is divided into three parts; the first relates to proportions, the second to geometry and the third, to algebra. Each of the three parts begins with definitions and demonstrations credited to the Arabs, then Leonardo considers six questions, three simple and three complex, giving solutions for them.

Leonardo, in 1228, gave a second edition of the Liber Abbaci which he dedicated to Michel Scott, astrologer to the Emperor Frederic II and author of many scientific works. Copies of this edition exist today. Leonardo profusely illustrated and strongly advocated the Hindu-Arabic system in this work. He gave an extensive discussion of the Rule of False Position and the Rule of Three. Leonardo did not use a general method in problem solving; each problem was solved independently of the others. In the solution of a problem he not only considered the problem as it might occur, but considered all of the variations of the question, even those that were not reasonable.

In the Liber Abbaci, Leonardo states and gives the solution to the famous Rabbit Problem [1, Vol. 1, p. 285]. A pair of rabbits are placed in a pen to find out how many offspring will be produced by this pair in one year if each pair of rabbits gives birth to a new pair of rabbits each month starting with the second month of its life; it is assured that deaths do not occur.

Leonardo traces the progress of the rabbits: The first pair has offspring in the first month: thus two pair. The second month there are three pair, the first reproducing in this month. In the third month there are five pair. Continuing in this manner through the twelve months. Leonardo gives the following table:

0
Pairs
1
First Month
2
Second Month
3
Third Month
5
Fourth Month
8
Fifth Month
13
Sixth Month
21
Seventh Month
34
Eighth Month
55
Ninth Month
89
Tenth Month
144
Eleventh Month
233
Twelfth Month
377

It is this sequence of numbers, $1, 2, 3, 5, 8, 13, \cdots$, that gives rise to the Fibonacci Sequence.

Of the many problems of elementary nature in the Liber Abbaci, the following are given as examples.

Seven old women are traveling to Rome and each has seven mules. On each mule there are seven sacks; in each sack there are seven loaves of bread: in each loaf there are seven knives; and each knife has seven sheaths. How many in all are going to Rome?

A man went into an orchard which had seven gates; and there took a certain number of apples. When he left the orchard he gave the first guard half the apples he had and one apple more. To the second he gave half the remaining apples and one apple more. He did the same in the case of each of the remaining five guards, and left the orchard with one apple. How many apples did he gather in the orchard?

A certain man puts one denarius at such a rate that in five years he has two denarii and in every five years thereafter the money doubles. How many denarii would he gain from this one denarius in 100 years?

A certain king sent thirty men into his orchard to plant trees. If they could set out a thousand trees in nine days, in how many days would thirty-six men set out four thousand four hundred trees?

Many readers will recognize these problems.

In 1220, Leonardo wrote Practica Geometriae, which he dedicated to Master Dominique, a person of whom there is no record. In this work Leonardo systematized the subject matter of practical geometry with a specialization in measurements of bodies. He included some algebra and trigonometry, square and cube roots, proportions and indeterminate problems. The use of a surveying instrument called the quadrans is included. The work is skillfully done with Euclidean rigor and some originality.

Leonardo's reputation grew and from his writings it can be seen that he had a vast range of knowledge concerning Arabian mathematics and mathematics of antiquity, especially Greek. His treatment shows much originality, completeness and rigor. It is especially noted that his writings did not contain the mysticism of numerology and astrology that were so prevalent in the writing of his day.

Because of Leonardo's great reputation, the Emperor Frederick II, when in Pisa (1225), held a sort of mathematical tournament to test Leonardo's skill. The competitors were informed beforehand of the questions to be asked, some or all of which were composed by Johannes of Palermo [1, Vol. II, p. 227], who was one of Frederick's staff. This is the first case in the history of mathematics that one meets with an instance of these challenges to solve particular problems which were so common in the sixteenth and seventeenth centuries.

The first question propounded was to find a number of which the square when decreased or increased by 5 would remain a square (II)( ). The correct answer given by Leonardo was 41/12. The next question was to find by the methods used in the tenth book of Euclid a line whose length X should satisfy the equation $x^3 + 2x^2 + 10x - 20 = 0$. Leonardo showed by geometry that the problem was impossible, but gave an approximation of the root 1.368808 1075 $\cdots$, which is correct to nine places.

The third question was:

Three men possess a certain sum of money, their shares in the ratio 3:2:1. While making the division, they were surprised by a thief and each took what he could and fled. Later the first man gave up half of what he had, the second gave up one-third, and the third, one-sixth. The money given up was divided equally among them and then each man had the share to which he was entitled. What was the total sum? Leonardo showed that the problem was indeterminate and gave as one solution 47 which is the smallest sum.

The other competitors failed to solve any of these questions. Through the consideration of these problems and others similar to them, Leonardo was led to write his Liber Quadratorum (1225) [No. 1, Vol. II, p. 253] a brilliant and original work containing a well arranged collection of theorems from indeterminate analysis involving equations of the second degree such as $x^2 + 5 = y^2$, $x^2 - 5 = z^2$. This work has marked him as the outstanding mathematician between Diophantus and Fermat in this field.

Two or three works of Leonardo that are known are the Flos [1, Vol. II, p. 227] (blossom or flower), which contains the last two problems of the tournament; the first problem is found in the Liber Quadratorum, and a Letter to Magister Theodoris [1, Vol. II, p. 247], philosopher to Frederick II, relating to indeterminate analysis and to geometry. The last three works show clearly the genius and brilliance of Leonardo as a mathematician and were beyond the abilities of most contemporary scholars.

The works of Leonardo Fibonacci are available in some universities in the United States through B. Boncompagni, Scritte di Leonardo Pisano, Rome, (1857-1862) [1]. The first volume contains the Liber Abbaci and the second volume contains Patricia Geometriae, the Flos, Letter to Magestrum Theodorum, and Liber Quadratorum. A treatment of square numbers composed by Leonardo and addressed to the Emperor Frederick II seems to have been lost.

REFERENCE

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2. A. F. Horadam, "A Generalized Fibonacci Sequence," Amer. Math. Monthly 68 (1961), pp. 455–459.
3. E. Lucas, "Théorie Des Fonctions Numérique Simplement Périodiques," Amer. J. Math. 1 (1878), pp. 184–240.
4. I. Nivin and H. S. Zuckerman, An Introduction to the Theory of Numbers, Wiley, New York, 1960.