$$2X = -10^n + \sqrt{10^{2n} + 4\cdot 10^{2n} - 4\cdot 10^n}$$
$$= -10^n + \sqrt{5\cdot 10^{2n} - 4\cdot 10^n} \quad .$$

Again for very large values we may ignore $4\cdot 10^n$ in the expression under the square-root sign, so having, as $n \to \infty$,

$$2X \to -10^n + 10^n\sqrt{5} \quad ,$$

i.e.,

$$X \to \frac{10^n(\sqrt{5} - 1)}{2} \quad .$$

Hence

$$X/Y \to (\sqrt{5} - 1)/2, \qquad Y/X \to (\sqrt{5} + 1)/2 \quad .$$

Fibonacci again!

It may be noted that with $n = 6$, the greatest value of Y (giving the minimal $X:Y$ ratio) gives

$$569466\ 945388 = 945388^2 - 569466^2 \ .$$

And for this we have $Y/X = 1.6601 \cdots$.

[Continued from page 196.]

$$\sum_{j=0}^{r}\sum_{k=0}^{\infty} c_{j,k}\, x^j y^k F(x,y) = \sum_{m=0}^{r-1} x^m \sum_{j=0}^{m}\sum_{k=0}^{\infty} c_{j,k}\, y^k \sum_{n=0}^{\infty} a_{m-j,n}\, y^n \ .$$

It follows that $F(x,y)$ is rational in x, again contradicting (2).

Remark. We note that $a_{m,n}$ does satisfy recurrences of the type

[Continued on page 217.]