THE LAMBERT FUNCTION

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The sum of certain reciprocal Fibonacci series can be summed in terms of the so-called Lambert series or Lambert function:

$$L(z) = \sum_{n=1}^{\infty} \frac{z^n}{1 - z^n} = \sum_{n=1}^{\infty} T_n (z)^n \quad ,$$

where T_n is the number of divisors of N*. For example, let

$$\beta = \frac{1 - \sqrt{5}}{2} \quad ,$$

$$\sum_{k=1}^{\infty} \frac{1}{F_{2k}} = \sqrt{5}\left[L\left(\frac{3 - \sqrt{5}}{2}\right) - L\left(\frac{7 - 3\sqrt{5}}{2}\right)\right] = \sqrt{5}\,[L(\beta^2) - L(\beta^4)]$$

or to generalize:

$$\sum_{k=1}^{\infty} \frac{1}{F_{2km}} = \sqrt{5}\,[L(2m\beta) - L(4m\beta)] \quad ,$$

for an integer m, such that $m > 0$.

In this note, we tabulate the Lambert function for selected real values of z. The results are given in the table below. The calculations were made by machine evaluation. The graph of the approximation polynomial to L(z) is shown on the following page.

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*Konrad Knopp, Theory and Application of Infinite Series, Harper, New York.

z	L_z	$L_{(-z)}$
.95	19.7372	4.7378
.90	14.4885	3.1728
.85	10.6987	2.0953
.80	7.9593	1.3565
.75	5.9724	.8513
.70	4.5224	.5066
.65	3.4550	.2720
.60	2.6605	.1130
.55	2.0615	.0062
.50	1.6035	-.0645
.45	1.2482	-.1096
.40	.9687	-.1363
.35	.7464	-.1493
.30	.5667	-.1518
.25	.4211	-.1456
.20	.3017	-.1316
.15	.2035	-.1103
.10	.1223	-.0817
.05	.0553	-.0452
.00	.0000