Many popular-type math teasers are based on the concept that may be expressed symbolically as:

\[(X)(Y) = Y^2 - X^2\]

Examples are:

\[34 \ 68 = 68^2 - 34^2\]
\[216 \ 513 = 513^2 - 216^2\]

The true algebraical representation, of course, is:

\[10^nX + Y = Y^2 - X^2\]

\(Y\) having \(n\) digits including any initial zero. For example, with \(n = 6\), we have:

\[2230\ 047276 = 47276^2 - 2230^2\]

Working recently on such examples, it seemed interesting to determine the limiting minimal value of the ratio \(Y:X\), that is of \(Y/X\). This proved quite simple, the derivation being as follows:

For very large values of \(n\) we may take the maximum value of \(Y\) as being \(10^n\).

Hence we have

\[10^nX + 10^n = 10^{2n} - X^2\]

Solving for \(X\),
Again for very large values we may ignore $4 \cdot 10^n$ in the expression under the square-root sign, so having, as $n \to \infty$,

$$2X \to -10^n + 10^n \sqrt{5},$$

i.e.,

$$X \to \frac{10^n(\sqrt{5} - 1)}{2}.$$

Hence

$$\frac{X}{Y} \to (\sqrt{5} - 1)/2, \quad \frac{Y}{X} \to (\sqrt{5} + 1)/2.$$

Fibonacci again!

It may be noted that with $n = 6$, the greatest value of $Y$ (giving the minimal $X:Y$ ratio) gives

$$569466 \times 945388 = 945388^2 - 569466^2.$$

And for this we have $\frac{Y}{X} = 1.6601 \ldots$.

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$$\sum_{j=0}^{r-1} \sum_{k=0}^{m} c_{j,k} x^j y^k F(x,y) = \sum_{m=0}^{r-1} \sum_{j=0}^{m} \sum_{k=0}^{\infty} c_{j,k} x^j y^k \sum_{n=0}^{\infty} a_{m-j,n} y^n.$$

It follows that $F(x,y)$ is rational in $x$, again contradicting (2).

Remark. We note that $a_{m,n}$ does satisfy recurrences of the type

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