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The solution is then given by Eq. (1.8) as

$$(2.5) \quad H_n = C_{11}\alpha^n + C_{12}n\alpha^{n-1} + C_{21}\beta^n + C_{22}n\beta^{n-1}$$

with the C_{ij} given by Eq. (1.9). In practice, however, the C_{ij} are most easily found by solving the set of simultaneous equations derived by applying the initial values, H_0, H_1, H_2, H_3 , for $n = 0, 1, 2, 3$. The solution yields:

$$C_{11} = \frac{3 - \alpha}{5} H_0 + \frac{2\alpha - 1}{5} H_1 + \frac{2}{25} (1 - 2\alpha)$$

$$C_{12} = 1/5$$

$$C_{21} = \frac{2 + \alpha}{5} H_0 + \frac{1 - 2\alpha}{5} H_1 + \frac{2}{25} (2\alpha - 1)$$

$$C_{22} = 1/5$$

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(If $M_2 = 1$, i. e., there is only one cell in the second group, then it cannot exchange with both $A_{M_1}^1$ and A_1^3 . The rearrangements corresponding to this case are eliminated in (6) since it occurs when $k_1 = k_2 = 1$ and $G(-1) = 0$.)

The remainder of the proof follows the same procedure. Define $k_j = 1$ if $A_{M_j}^j$ and A_1^{j+1} exchange, $k_j = 0$ otherwise, $j = 3, \dots, N-1$. For each of 2^{N-1} possible values of $(k_1, k_2, \dots, k_{N-1})$ the number of distinct arrangements of the N groups combined is

$$(7) \quad G(M_1 - k_1) + G(M_N - k_{N-1}) \cdot \prod_{j=2}^{N-1} G(M_j - k_{j-1} - k_j).$$

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(where $q' = r(q'-1)$), which are the numbers $u(n; q, r)$ in the Tribonacci convolution triangle! See [4].

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The total number of distinct arrangements of the N groups combined is obtained by summing the expression in (7) over all possible values of $(k_1, k_2, \dots, k_{N-1})$, i. e., over the set S_{N-1} . But the total number of distinct arrangements is also equal to

$$G\left(\sum_{j=1}^N M_j\right).$$

The identity in (3) then follows from $G(n) = F(n+1)$.

